Math 311 Hwk 6

Show your work.

Problem 1 (10 points)**.** *An $n \times n$ matrix A is said to be idempotent if $A^2 = A$. Show that if λ is an eigenvalue of an idempotent matrix, then $\lambda = 0$ or $\lambda = 1$.*

Problem 2 (40 points)**.** *Prove the following for an $m \times n$ matrix A:*

(a). *If $\mathbf{x} \in \mathcal{N}(A^H A)$, then $A\mathbf{x}$ is in both $\mathcal{R}(A)$ and $\mathcal{N}(A^H)$.*

(b). $\mathcal{N}(A^H A) = \mathcal{N}(A)$.

(c). *A and $A^H A$ have the same rank.*

(d). *If A has linearly independent columns, then $A^H A$ is nonsingular.*

Problem 3 (30 points)**.** *A linear operator Q on $\mathbb{C}^n$ is called unitary if all of its column vectors are orthonormal.*

(a). *Prove that Q is unitary iff $Q^H Q = QQ^H = I$.*

(b). *Prove that if Q is unitary, then $\|Q\mathbf{x}\| = \|\mathbf{x}\|$ for all $\mathbf{x} \in V$.*

(c). *Prove that if Q is unitary, then $|det(Q)| = 1$. Is the converse true?*

Problem 4 (20 points)**.** *Let $V = C[0, 1]$ have the inner product*

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx.$$

Find the angle θ between the vectors x^2 and x^4.