

Math 311 Hwk 7

Show your work.

Problem 1 (20 points). Write a Matlab function called `ginv` that takes an $m \times n$ matrix A and returns its generalized inverse A^+ . You may use the built-in `svd` function.

Problem 2 (20 points). Let A be an $m \times n$ matrix. Prove that $\text{tr}(A^H A) = 0$ implies that $A = 0$.

Problem 3 (20 points). Show that $\langle A, B \rangle = \text{tr}(A^H B)$ is an inner product on $M_n(\mathbb{C})$.

Problem 4 (20 points). Let

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 6 & 3 & 0 \\ 6 & 3 & 0 \\ 2 & 5 & 4 \end{pmatrix}.$$

Using the singular value decomposition, find:

- (a). An orthonormal basis for $\mathcal{N}(A)$.
- (b). An orthonormal basis for $\mathcal{R}(A^H)$.
- (c). An orthonormal basis for $\mathcal{R}(A)$.
- (d). An orthonormal basis for $\mathcal{N}(A^H)$.

Problem 5 (20 points). Given

$$A = \begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

- (a). Use the generalized inverse to find a least squares solution to the system $A\mathbf{x} = \mathbf{b}$.
- (b). Find all solutions to the least squares problem $A\mathbf{x} = \mathbf{b}$.