Math 311 Extra Credit

Show your work.

Problem 1 (25 points). *Find the Fourier series expansion of* $f(x) = |x|$ *on the interval* $[\pi, \pi]$. *Use your answer to then find the Fourier series expansion of* $f(x) = 3|x| - 4$ *on the same interval.*

Problem 2 (25 points). *Write a Matlab function that finds* $\langle x^n, \sin kx \rangle$ *and* $\langle x^n, \cos kx \rangle$, *where*

$$\langle f(x), g(x) \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)\,dx.$$

Problem 3 (50 points). *Use the function in the previous problem to find the (truncated) Fourier series of a given polynomial* $p(x)$ *on the interval* $[-\pi, \pi]$. *As example, use* $p(x) = 7x^3 + 5x^2 + 3x - 1$. *Expand the series for several values of* n. *Plot the original curve and the approximate curves. For* 20 *additional bonus points, compute the errors for different values of* n *as well. Hint, the Fourier series is the least squares projection.*