## Math 316 Hwk 8

Problem 1. Minimize $x_{1}^{2}+2 x_{1} x_{2}+3 x_{2}^{2}+4 x_{1}+5 x_{2}+6 x_{3}$ subject to the constraints $x_{1}+2 x_{2}=3$ and $4 x_{1}+5 x_{3}=6$.

Problem 2. Maximize $4 x_{1}+x_{2}^{2}$ subject to the constraint $x_{1}^{2}+x_{2}^{2}=9$.
Problem 3. Find all solutions to the problem: Maximize $\mathbf{x}^{T} A \mathbf{x}$ subject to $\|\mathrm{x}\|^{2}=1$, when

$$
A=\left[\begin{array}{ll}
3 & 4 \\
0 & 3
\end{array}\right]
$$

Problem 4. Minimize $f(\mathbf{x})$ subject to $C \mathbf{x}=\mathbf{d}$ where

$$
f(\mathbf{x})=\frac{1}{2}\|A \mathbf{x}-\mathbf{b}\|^{2},
$$

$A \in \mathbb{R}^{m \times n}, C \in \mathbb{R}^{p \times n}$ are both of full rank, $m>n$ and $p<n$.
Problem 5. Suppose that $\mathbf{x}^{*}$ is the minimum of the function $f(\mathbf{x})$ subject to the constraint $h(\mathbf{x})=0$, where $f$ and $h$ are maps from $\mathbb{R}^{2}$ into $\mathbb{R}$. If for $\mathbf{x}=\left(x_{1}, x_{2}\right)$ we have $\operatorname{Df}(\mathbf{x})=\left[\begin{array}{ll}x_{1} & x_{1}+4\end{array}\right]$ and $\operatorname{Dh}\left(\mathbf{x}^{*}\right)=\left[\begin{array}{ll}1 & 4\end{array}\right]$, find $D f\left(\mathbf{x}^{*}\right)$.

