## Math 316 Hwk 8

**Problem 1.** Minimize  $x_1^2 + 2x_1x_2 + 3x_2^2 + 4x_1 + 5x_2 + 6x_3$  subject to the constraints  $x_1 + 2x_2 = 3$  and  $4x_1 + 5x_3 = 6$ .

**Problem 2.** Maximize  $4x_1 + x_2^2$  subject to the constraint  $x_1^2 + x_2^2 = 9$ .

**Problem 3.** Find all solutions to the problem: Maximize  $\mathbf{x}^T A \mathbf{x}$  subject to  $\|\mathbf{x}\|^2 = 1$ , when

$$A = \begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}.$$

**Problem 4.** Minimize  $f(\mathbf{x})$  subject to  $C\mathbf{x} = \mathbf{d}$  where

$$f(\mathbf{x}) = \frac{1}{2} \|A\mathbf{x} - \mathbf{b}\|^2,$$

 $A \in \mathbb{R}^{m \times n}$ ,  $C \in \mathbb{R}^{p \times n}$  are both of full rank, m > n and p < n.

**Problem 5.** Suppose that  $\mathbf{x}^*$  is the minimum of the function  $f(\mathbf{x})$  subject to the constraint  $h(\mathbf{x}) = 0$ , where f and h are maps from  $\mathbb{R}^2$  into  $\mathbb{R}$ . If for  $\mathbf{x} = (x_1, x_2)$  we have  $Df(\mathbf{x}) = \begin{bmatrix} x_1 & x_1 + 4 \end{bmatrix}$  and  $Dh(\mathbf{x}^*) = \begin{bmatrix} 1 & 4 \end{bmatrix}$ , find  $Df(\mathbf{x}^*)$ .