

Dr. Humpherys: Math 332 Section 1 Exam 1.

Your textbook and class notes are the only resources you may use.
No help from anyone, no internet, no other books, etc. Show your work.

Problem 1. Simplify the following into the form $a + ib$, where $a, b \in \mathbb{R}$.

(i). $\sqrt{1 + \sqrt{i}}$

(ii). $\sqrt{1 + i}$

(iii). $\sqrt{\sqrt{-i}}$

Problem 2. If $|z| = 1$, and $a, b \in \mathbb{C}$, prove that

$$\left| \frac{az + b}{\overline{bz + a}} \right| = 1.$$

Problem 3. Let f be analytic on the open, connected subset A of \mathbb{C} . Define $g(z) = \overline{f(\bar{z})}$ on the set

$$\bar{A} = \{z \mid \bar{z} \in A\}.$$

Prove that g is analytic on \bar{A} .

Problem 4. Prove Theorem 1(iii) on page 61 of the text. Be exactly correct.

Problem 5. Prove the identity

$$\sin \frac{\pi}{n} \sin \frac{2\pi}{n} \cdots \sin \frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}.$$

Hint: The given product can be written as $1/2^{n-1}$ times the product of the nonzero roots of the polynomial $(1 - z)^n - 1$.