Project Ideas

Problem 1 (N-Body Problems). *Examine attracting or repelling models from semi-classical physics and planetary motion. Projects may include examining the chaotic dynamics of the 3-body problem (sensitivity to initial data) or statistical properties of the n-body problem, .*

Problem 2 (Predator-Prey Model). *Let $R(t)$ be the number of rabbits and $F(t)$ be the number of foxes. Consider the system*

$$\frac{dR(t)}{dt} = aR(t) - bR(t)F(t)$$

$$\frac{dF(t)}{dt} = cR(t)F(t) - dF(t),$$

where a is the natural growth rate of the rabbits in the absence of prediation, b is the death rate per encounter of rabbits due to predation, c is the growth rate of foxes in the presence of rabbits, and d is the natural death rate of foxes in the absence of rabbits.

Problem 3 (Lorentz's Strange Attractor). *Explore the sensitivity to initial data for the system*

$$\frac{dx}{dt} = \sigma(y(t) - x(t))$$

$$\frac{dy}{dt} = \rho x(t) - y(t) - x(t)z(t)$$

$$\frac{dz}{dt} = -\beta z(t) + x(t)y(t).$$

This is a model used in weather prediction. For starters try the parameters $\sigma = 10$, $\rho = 28$, $\beta = 8/3$, and initial data $(0, 1, 0)$.

Problem 4 (Traveling Waves). *Explore any of the following models: Korteweg-de Vries (Tsunamis), Shallow Water Equations (Dam Breaking), Burger's Equation (Shock Waves), Nonlinear Schrdinger Equation (Fiber Optics)*

Problem 5 (Missile and Airplane Problem). *Explore the dynamics of a guided missile chasing after an airplane.*

Problem 6 (Coupled Oscillators). *Explore the dynamics of an array of coupled harmonic oscillators*

Problem 7 (Oscillators). *Study effects of forcing in spring-mass or RLC circuits. Examine the effects of resonance.*

Problem 8 (Near equilibrium dynamics). *Explore phase diagrams for different kinds of critical points. Plot saddle points, spirals, nodes, and degenerate dynamics.*

Problem 9 (Bead and a Rotating Hoop). *Explore the bifurcation of a bead connected to a rotating and frictionless hoop.*

Problem 10 (Trajectories with Wind Resistance). *Explore the dynamics of a projectile in the presence of wind resistance.*