Math 411: Optimization

Newton's Method

$$x_{n+1} = x_n - Df(x_n)^{-1} f(x_n).$$

Instead of actually computing the inverse, we solve the equation for y_n

$$Df(x_n) y_n = -f(x_n).$$

Then set

$$x_{n+1} = x_n + y_n.$$

Broyden's Method

Initialize with

$$A_0 = Df(x_0)$$

Set

$$x_1 = x_0 - A_0^{-1} f(x_0).$$

Then appoximate $Df(x_1)$ with A_1, where

$$A_1 = A_0 + \frac{f(x_1) - f(x_0) - A_0(x_1 - x_0)}{\|x_1 - x_0\|^2}(x_1 - x_0)^T.$$

Then compute x_2 via

$$x_2 = x_1 - A_1^{-1} f(x_1).$$

Repeating iteratively, we have

$$A_n = A_{n-1} + \frac{f(x_n) - f(x_{n-1}) - A_{n-1}(x_n - x_{n-1})}{\|x_n - x_{n-1}\|^2}(x_n - x_{n-1})^T,$$

and

$$x_{n+1} = x_n - A_n^{-1} f(x_n).$$

Rather than inverting A_n each time, we use the following rank-one update

$$A_n^{-1} = A_{n-1}^{-1} + \frac{((x_n - x_{n-1}) - A_{n-1}^{-1}(f(x_n) - f(x_{n-1})))(x_n - x_{n-1})^T A_{n-1}^{-1}}{(x_n - x_{n-1})^T A_{n-1}^{-1}(f(x_n) - f(x_{n-1}))}.$$