# Math 411 Hwk 2

**Problem 1** (25 points)**.** *Let $A$ be a nonsingular $n \times n$ matrix, $B$ a $n \times k$ matrix, $C$ a nonsingular $k \times k$ matrix, and $D$ a $k \times n$ matrix. Prove the following matrix identity, which is called the Sherman-Morrison-Woodbury formula:*

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}.$$

**Problem 2** (25 points)**.** *Use the Sherman-Morrison-Woodbury formula to show that the inverse of the rank-one update in Broyden's method,*

$$A_n = A_{n-1} + \frac{f(x_n) - f(x_{n-1}) - A_{n-1}(x_n - x_{n-1})}{\|x_n - x_{n-1}\|^2}(x_n - x_{n-1})^T,$$

*can be simplified to the equation*

$$A_n^{-1} = A_{n-1}^{-1} + \frac{((x_n - x_{n-1}) - A_{n-1}^{-1}(f(x_n) - f(x_{n-1})))(x_n - x_{n-1})^T A_{n-1}^{-1}}{(x_n - x_{n-1})^T A_{n-1}^{-1}(f(x_n) - f(x_{n-1}))}.$$

**Problem 3** (50 points)**.** *The Vandermonde matrix is defined as*

$$V_n = \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix}.$$

*Prove that*

$$det(V_n) = \prod_{i<j}(x_j - x_i).$$

*Hint: Row reduce the transpose. Subtract $x_k$ times the $(k-1)^{\texttt{th}}$ row from the $k^{\texttt{th}}$ row for $k = 1, \ldots, n$. Then factor out all the $(x_k - x_0)$ terms to reduce the problem. Then proceed recursively.*