Math 411 Hwk 6

Problem 1. Write a Matlab function that uses finite differences to solve the boundary-value problem

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = f(x), \quad a \le x \le b,$$

satisfying $y(a) = \alpha$ and $y(b) = \beta$. Assume that the mesh is evenly spaced. Call the function mybvp(p,q,r,f,interval,boundary). Make sure that your program can handle very fine grids (use sparse solver).

Problem 2. Use the Galerkin method to numerically solve the problem

$$y'' + y = -1, \quad 0 \le x \le 1,$$

where y(0) = y(1) = 0. Compare your numerical solution with the exact one given by

$$y(x) = -1 + \cos x + \frac{1 - \cos 1}{\sin 1} \sin x.$$

Repeat this for several mesh sizes. What conclusions can you draw?