

Math 635 Hwk 4

Problem 1. Write a Matlab function that uses finite differences to solve the boundary-value problem

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = f(x), \quad a \leq x \leq b,$$

satisfying $y(a) = \alpha$ and $y(b) = \beta$. Assume that the mesh is evenly spaced. Call the function `mybvp(p,q,r,f, interval, boundary)`. Make sure that your program can handle very fine grids (use sparse solver).

Problem 2. Use the Galerkin method to numerically solve the problem

$$y'' + y = -1, \quad 0 \leq x \leq 1,$$

where $y(0) = y(1) = 0$. Compare your numerical solution with the exact one given by

$$y(x) = -1 + \cos x + \frac{1 - \cos 1}{\sin 1} \sin x.$$

Repeat this for several mesh sizes. What conclusions can you draw?