Homework 15, due October 9

(1) (a) Divide 3^{10203} by 101. What is the remainder?
    (b) Find a primitive root modulo 97.
(2) Prove or provide a counterexample for each.
    (a) $\gcd(n, \varphi(n)) > 1$.
    (b) If $d|m$, then $\varphi(d)|\varphi(m)$.
    (c) If the same primes divide m and n, then $n\varphi(m) = m\varphi(n)$.
(3) (Page 108, problem 26) Let $p \equiv 3 \pmod 4$ be prime. Show that $x^2 \equiv -1 \pmod p$ has no solutions. (Hint: Suppose x exists. Raise both sides to some power and find a contradiction.)