Homework 3, due September 9

- (1) Compute gcd(455, 1235) and gcd(2415, 6909) by hand. Find integers x, y, z, w such that 455x + 1235y = 65 and 2415z + 6909w = 63.
- (2) The Fibonacci numbers are given by  $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$ . True or false: Any two consecutive Fibonacci numbers are relatively prime. If true, prove it. If false, give a counterexample.
- (3) Let a and b be integers. Using the definition of congruence modulo n, prove that  $a \equiv b \pmod{n}$  if and only if  $a^2 + b^2 \equiv 2ab \pmod{n^2}$ .