Math 487, Midterm Exam #2 Study Guide

General information

(1) The exam will cover Chapters 3 and 4 (through section 4.2). Books and notes will not be allowed. Testing center calculators will be provided.
(2) WARNING: this study guide is not meant to be exhaustive. Just because something is not on the study guide does not mean it will not be on the exam.

Basics

(1) Concepts from the first midterm exam: rings, groups, fields, integral domains, divisibility, prime, gcd, division algorithm, Euclidean algorithm, LCM, fundamental theorem of arithmetic, congruences, $\mathbb{Z}_n$, Euler φ-function, units, Fermat's little theorem, order of elements, primitive roots, Chinese remainder theorem, quadratic residues, Legendre and Jacobi symbols, quadratic reciprocity
(2) Definitions:
- Riemann zeta function
- Fermat numbers
- Mersenne numbers
- Perfect numbers
- Fibonacci numbers
- Golden section
- Quadratic forms
- Positive definite quadratic forms
- Pythagorean triple
- Method of infinite descent
- Dirichlet character
- Dirichlet L-series
- Möbius function $\mu(n)$
- von Mangoldt function $\Lambda(n)$
- Twin primes
- Arithmetic functions: $\tau(n), \sigma(n), \sigma_k(n)$
- Multiplicative function
- Binomial coefficient
- Big $\mathcal{O}$, little o, same order of magnitude, asymptotically equal
- Prime counting function $\pi(x)$

Theorems you should know and be able to use

- There are infinitely many primes.
- Euler product expansion of zeta function
- Continued fraction expansion of real numbers
- Dirichlet's theorem on primes in arithmetic progressions
- Fermat's two-square theorem
- Lagrange's four-square theorem
- Properties of Dirichlet characters: Lemma 3.3.1, Lemma 3.3.3, Lemma 3.3.4, Corollary 3.3.1, Theorem 3.3.2
- Euler product representation of L-series
- Theorem 3.6.1
- Möbius inversion formula
- Prime number theorem
- Theorem 4.1.2
- Binomial theorem
- Chebychev's estimate
- Combinatorial proofs for binomial coefficients and Fibonacci numbers

Things you should be able to prove (and use)

- Binet's formula for Fibonacci numbers
- Theorem 3.3.1 (orthogonality relations for Dirichlet characters)
- Theorem 3.6.3