

EXAM 1

RED

MATH 302 - Advanced Engineering Mathematics I

September 28 - October 2, 2007

Name: _____

Instructor: _____

General Directions

1. For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
2. Simplify your answers.
3. Calculators are not allowed.
4. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
5. Please do not talk about the test with other students until after the last day to take the exam.

Question	Possible Points	Points Earned
MC	(50)	
11	(10)	
12	(10)	
13	(10)	
14	(10)	
15	(10)	
Total	(100)	

Part I: Multiple Choice Mark the correct answer on the bubble sheet provided.

1. Find the projection of the vector $\vec{v} = \langle 0, -2, 2 \rangle$ onto $\vec{u} = \langle 1, 1, 4 \rangle$

- (a) $\langle \frac{1}{3}, \frac{1}{3}, \frac{4}{3} \rangle$ (b) $\langle 3, 3, 12 \rangle$ (c) $\langle 0, -\frac{3}{2}, \frac{3}{2} \rangle$
(d) $\langle 0, -\frac{8}{3}, \frac{8}{3} \rangle$ (e) $\frac{\sqrt{18}}{3}$ (f) $\frac{3\sqrt{8}}{4}$

Answer: a

2. Find the area of a triangle with vertices $(1, 0, 2)$, $(3, 2, 1)$, and $(2, 1, 3)$.

- (a) 3 (b) 9 (c) $3\sqrt{2}$
(d) $\frac{3}{2}$ (e) $\frac{9}{2}$ (f) $\frac{3}{2}\sqrt{2}$

Answer: a

3. Find the vector of length 6 in the direction of $\langle 1, 2, -2 \rangle$.

- (a) $\langle \frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \rangle$ (b) $\langle 2, 4, -4 \rangle$ (c) $\langle \frac{1}{6}, \frac{1}{3}, -\frac{1}{3} \rangle$
(d) $\langle 6, 12, -12 \rangle$ (e) $\langle \frac{5}{6}, \frac{10}{6}, -\frac{10}{6} \rangle$ (f) None of these.

Answer: b

4. Which of the following represents the equation of the line through $(2, -1, 4)$ perpendicular to the plane $x + 3y - z = 6$.

- i. $\vec{r}(t) = \langle 2 + t, -1 + 3t, 4 - t \rangle$
ii. $\vec{r}(t) = \langle 1 + 2t, 3 - t, -1 + 4t \rangle$
iii. $\frac{x-1}{2} = 3 - y = \frac{z+1}{4}$
iv. $x - 2 = \frac{y+1}{3} = 4 - z$

- (a) both (i) and (iii). (b) both (i) and (iv) (c) both (ii) and (iii).
(d) both (ii) and (iv). (e) (i) only (f) (ii) only

Answer: b

5. If $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 2 & 0 & 2 \end{bmatrix}$, what is the entry in the third row, second column of A^3 ?

- (a) 1 (b) 6 (c) 17
(d) 20 (e) 23 (f) 30

Answer: b

6. Let $A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}$. Which matrix is **not** a linear combination of A_1 and A_2 ?

(a) $\begin{bmatrix} 3 & 5 \\ -2 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 6 & 4 \\ 2 & 0 \end{bmatrix}$

Answer: d

7. Which matrix a)-e) is **not** an elementary matrix?

(a) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(f) None of the above are elementary matrices?

Answer: a

8. If

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix},$$

then $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}.$$

The entry in the third row and third column of U is

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) 2

(e) $\frac{5}{2}$

(f) 3

Answer: f

9. Find the rank of the following matrix.

$$\begin{pmatrix} 1 & 2 & 7 & 1 & 3 \\ 3 & 0 & 9 & 3 & 3 \\ 1 & 1 & 5 & 1 & 2 \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix}$$

- (a) 1 (b) 2 (c) 3
(d) -4 (e) -1 (f) The matrix is not semisimple so it has no rank.

Answer: b

10. Solve the following system for x, y, z if possible and find $x + y + z$ if there is a solution.

$$\begin{aligned} x + y - 2z &= -5 \\ x - z &= -4 \\ -2x - y + 4z &= 12 \end{aligned}$$

- (a) 2 (b) 3 (c) 4
(d) 1 (e) -3 (f) There is no solution. The system is inconsistent.

Answer: c

11. Solve the following system for x, y, z if possible and find $3y - 2x - 2z$ if there is a solution.

$$\begin{aligned} x + y - 4z &= 1 \\ x - 2z &= -1 \\ -2x - y + 6z &= 0 \end{aligned}$$

- (a) 8 (b) -5 (c) 3
(d) 4 (e) -2 (f) There is no solution. The system is inconsistent.

Answer: f

12. Solve the following system for x, y, z if possible and find $3y + x - z$ if there is a solution.

$$\begin{aligned}x + y - 4z &= -1 \\x - 2z &= -2 \\-2x - y + 6z &= -4\end{aligned}$$

(a) 7

(b) 6

(c) 5

(d) -4

(e) -3

(f) There is no solution.
The system is inconsistent.

Answer: f

Part II: Free Response

13. Let R be a rectangular box with dimensions $2 \times 3 \times 4$. Find the angle between a short edge and a long diagonal of the box, meeting at a common vertex. (Note: A long diagonal is a diagonal that passes through the interior of the box, and is not contained in a face of the box.)

Solution. The two vectors are $\langle 2, 0, 0 \rangle$ and $\langle 2, -1, 4 \rangle$, then apply the angle formula we got $\theta = \arccos \frac{2}{\sqrt{29}}$.

14. Let P be the plane through the point $(1, 2, -1)$ with normal vector $\langle 4, -2, 3 \rangle$. Let L be the line through the origin perpendicular to the z -axis and the vector $\langle 1, -1, 0 \rangle$.

- (a) Find the equation of the plane P .
- (b) Find the equation of the line L .
- (c) Find the point q that is the intersection of P and L , or show that no such point exists.

Solution.

- (a) The equation of the plane P is

$$4(x - 1) - 2(y - 2) + 3(z + 1) = 0.$$

- (b) Because L is perpendicular to $\langle 1, -1, 0 \rangle$ and the z -axis, then its direction vector \vec{d} can be obtained by taking the cross product of $\langle 1, -1, 0 \rangle$, and $\langle 0, 0, 1 \rangle$:

$$\vec{d} = \begin{bmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = \langle 1, 1, 0 \rangle$$

then the equation of the line L is

$$L = \begin{cases} x = t \\ y = t \\ z = 0 \end{cases}$$

- (c) To find out the intersection point p of P and L , just plug the equation of L into P ,

$$4(t - 1) - 2(t - 2) + 3 = 0,$$

with simple calculation, $t = -\frac{3}{2}$, then $p = (-\frac{3}{2}, -\frac{3}{2}, 0)$.

15. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 1 & 8 \\ 3 & -1 & 4 & 2 & -2 \\ 2 & -1 & 3 & 3 & -4 \end{bmatrix}.$$

Find

- (a) the rank and nullity of A .
- (b) A basis for $\text{Col}(A)$.
- (c) A basis for $\text{Row}(A)$.
- (d) A basis for $\text{Null}(A)$.

Solution. With simple calculation, the reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & 0 & 8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) The rank of A is number of columns have leading 1, which is 3. The nullity = number of columns - rank = $6 - 3 = 3$.
- (b) Basis for $\text{Col}(A)$ is

$$\{ \langle 1, 0, 3, 2 \rangle^T, \langle 0, 1, -1, -1 \rangle^T, \langle -1, 1, 2, 3 \rangle^T \}$$

- (c) Basis for $\text{Row}(A)$ is

$$\{ \langle 1, 0, 1, -1, 2 \rangle, \langle 0, 1, -1, 1, 8 \rangle, \langle 3, -1, 4, 2, -2 \rangle^T \}$$

- (d) Basis for $\text{Null}(A)$ is

$$\{ \langle -1, 1, 1, 0, 0 \rangle, \langle -2, -8, 0, 0, 1 \rangle \}$$

16. If A and B are symmetric, is AB always symmetric? Show why it is, or give an example if it is not.

proof. NO! The example is let

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix},$$

then

$$AB = \begin{bmatrix} 2 & 2 \\ 5 & 2 \end{bmatrix}$$

which is not symmetric.

17. If

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

find its inverse.

Solution. The inverse is

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & -1 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

18. Suppose \mathbf{u}, \mathbf{v} are non zero vectors and $\{\mathbf{u}, \mathbf{v}\}$ is dependent. Show that one of these vectors is a multiple of the other.

Proof. Because \mathbf{u}, \mathbf{v} are dependent, then there exists non zero constants a, b such that $a\mathbf{u} + b\mathbf{v} = \mathbf{0}$. Then solve for \mathbf{u} we got that

$$\mathbf{u} = -\frac{b}{a}\mathbf{v},$$

The proof is complete.

19. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set, is it true or false that $\{\mathbf{u} + \mathbf{v}, \mathbf{v} + \mathbf{w}, \mathbf{u} + \mathbf{w}\}$ is linearly independent. If it is true, you must prove it is true. If it is false, you must give an example or some reason why it is false.

Proof. Let a, b, c be content such that

$$a(\mathbf{u} + \mathbf{v}) + b(\mathbf{v} + \mathbf{w}) + c(\mathbf{u} + \mathbf{w}) = 0,$$

simplify above equation we got

$$(a + c)\mathbf{u} + (a + b)\mathbf{v} + (b + c)\mathbf{w} = 0,$$

since $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set, then

$$a + c = a + b = b + c = 0, \Rightarrow a = b = c = 0,$$

Thus $\{\mathbf{u} + \mathbf{v}, \mathbf{v} + \mathbf{w}, \mathbf{u} + \mathbf{w}\}$ is linearly independent.

20. Here are some vectors:

$$\begin{pmatrix} 6 \\ 0 \\ 12 \\ 6 \end{pmatrix} \begin{pmatrix} 12 \\ 6 \\ 0 \\ 6 \end{pmatrix} \begin{pmatrix} 1 \\ -10 \\ 42 \\ 11 \end{pmatrix} \begin{pmatrix} 6 \\ 12 \\ 6 \\ 6 \end{pmatrix}$$

Determine whether they are independent or dependent and if they are dependent, give a dependence relation.

Solution. $\begin{pmatrix} 6 & 12 & 1 & 6 \\ 0 & 6 & -10 & 12 \\ 12 & 0 & 42 & 6 \\ 6 & 6 & 11 & 6 \end{pmatrix}$, row echelon form: $\begin{pmatrix} 1 & 0 & \frac{7}{2} & 0 \\ 0 & 1 & -\frac{5}{3} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and so a

dependence relation is $\frac{7}{2} \begin{pmatrix} 6 \\ 0 \\ 12 \\ 6 \end{pmatrix} - \frac{5}{3} \begin{pmatrix} 12 \\ 6 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -10 \\ 42 \\ 11 \end{pmatrix}$