Final Review Topics

1 Chapter 1 and 2 Functions

Chapter 1 are linear functions. If you have troubles with the algebra for Exponential/Logarithmic functions review chapter 2.

2 Differential Calculus

2.1 Limits

\[ \lim_{x \to a} f(x) = L \]

Read as “The limit as x approaches a of the function f(x) is L.”

- If \( f(x) \) is continuous at \( x = a \) then \( \lim_{x \to a} f(x) = f(a) \).

- A limit exists if and only if \( \lim_{x \to a^{-}} f(x) = \lim_{x \to a^{+}} f(x) \).

- If \( f(a) = 0 \) for \( \lim_{x \to a} f(x) \) then you must try to rewrite \( f(x) \) (factoring, multiplying by conjugate etc.) to get a continuous function at \( x = a \) to be able to evaluate the limit.

- Review limits at infinity
2.2 Continuity

\( f(x) \) is continuous at \( x = a \) if:

1. \( f(a) \) is defined
2. \( \lim_{x \to a} f(x) \) exists
3. \( \lim_{x \to a} f(x) = f(a) \)

2.3 Derivatives

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

- The derivative is the instantaneous rate of change of a function at a specified \( x \) value.
- \( f'(a) \) is the slope of the line tangent to \( f(x) \) at \( x = a \). The equation for the tangent line is

\[ y - f(a) = f'(a)(x - a) \]

2.3.1 Derivative Rules

- Power rule: \( \frac{d}{dx} (x^n) = nx^{n-1} \)
- Product rule: \( \frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \)
- Quotient rule: \( \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \)
- Chain rule: \( \frac{d}{dx} (f(g(x))) = f'(g(x))g'(x) \)

Be sure to have memorized the derivatives for individual functions (exponential, logarithmic, etc.)
2.3.2 Graphing a function using derivatives

- Find the intervals of increase and decrease from $f'(x)$ (Critical points are points in the domain of $f$ where $f'(x) = 0$ or is undefined)
- Find the intervals of concavity from $f''(x)$ (Inflection points are where the function changes concavity)

2.3.3 Maximums and minimums

- A critical point is a:
  - relative max if the derivative to the left of the critical point is positive and the derivative to the right of the critical point is negative.
  - relative min if the derivative to the left of the critical point is negative and the derivative to the right of the critical point is positive.
- The absolute/global max or min is the $y$ value that is greater or smaller respectively than every other $y$ value of the function.
  - We use the Extreme Value Theorem for absolute extrema: “A function $f$ that is continuous on a closed interval $[a, b]$ will have both an absolute maximum and an absolute minimum on the interval.”
  - p. 328 describes how to find absolute extrema.

Be sure to review section 6.2 for word problems involving applications of Extrema.

2.3.4 Implicit Differentiation

This is NOT a multivariable derivative situation. You use implicit differentiation when $x$ is an independent variable and $y$ is dependent on $x$ but you cannot solve for $y$ in terms of $x$.

- Rule: When you take the derivative of a $y$ variable you must multiply that term on by $\frac{dy}{dx}$.
- Warning: All previous derivative rules still hold.
3 Sequences and Series

3.1 Geometric sequence

- geometric series \( a_n = ar^{n-1} \) where \( a \) is the first term in the sequence and \( r \) is the common ratio.
- Know all of the formulas involving geometric sequences
- Review the fact that the sum of an infinite series can converge to a finite number :) 

3.2 Taylor Polynomial

\[
P(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n
\]

4 Integral Calculus

4.1 Antiderivatives

An antiderivative of \( f(x) \) is a function \( F(x) \) such that \( F'(x) = f(x) \).

4.2 Indefinite Integral

And antiderivative is a synonym for an indefinite integral

\[
F(x) = \int f(x)dx
\]

- Substitution: In order to choose what to make your \( u \) focus on the most complicated part of the integral.
- By parts: If you are using the table way just remember that the rules change for \( \log \) functions. Otherwise use

\[
\int uv = uv - \int vdu
\]
4.3 Definite Integral

The Fundamental Theorem of Calculus: If $F(x)$ is the antiderivative of $f(x)$ then:

$$\int_{a}^{b} f(x) dx = F(b) - F(a)$$

- $\int_{a}^{b} f(x) dx$ represents the area underneath the curve $f(x)$ on the interval $[a, b]$.
- Review area between two curves.

4.4 Volume, Average value, Improper integrals

- Volume of revolution: $\int_{a}^{b} \pi (f(x))^2 \, dx$
- Average value: $\frac{1}{b-a} \int_{a}^{b} f(x) dx$
- Improper integrals are where one of the bounds is either infinity or makes the function undefined (ex $x = 0$ with $\ln(x)$). Review how to completely these integrals.

5 Multi-variable Calculus

5.1 Differential Multi-Variable Calculus

- A partial derivative with respect to a specified variable is taking a normal derivative only considering that one variable and calling everything else a constant. (ex $f_x$ would be treating $x$ as a variable and $y$ as a constant.)
- Optimization (max and mins) still requires us to find critical points
- A critical point $(a, b)$ is where $f_x(a, b) = 0$ and $f_y(a, b) = 0$ at the same time.

$$D(x, y) = f_{xx}(x, y) \cdot f_{yy}(x, y) - [f_{xy}(x, y)]^2$$
1. $D(a, b) > 0$ and $f_{xx}(a, b) > 0$ gives us a minimum at $(a, b)$
2. $D(a, b) > 0$ and $f_{xx}(a, b) < 0$ gives us a maximum at $(a, b)$
3. $D(a, b) < 0$ gives us a saddle at $(a, b)$
4. $D = 0$ gives no information

- Lagrange Multipliers: $f(x, y)$ is what we want to optimize with a restriction of $g(x, y) = 0$.

\[ F(x, y, \lambda) = f(x, y) - \lambda g(x, y) \]

Find the $x$ and $y$ value that makes $F_x$, $F_y$, and $F_\lambda$ equal to zero.

### 5.2 Integral Multi-Variable Calculus

This finds the volume under a function over a certain region.

- All of the rules for normal integration still hold (substitution, by parts)
- Be careful what variable you are taking the antiderivative with respect to based on what order the $dy$ and $dx$ are given.

### 6 Differential Equations

- Separable: Get all of one variable on one side of the equation and all of the other on the opposite side. Then integrate both sides.

- Linear First-Order:

\[ \frac{dy}{dx} + P(x)y = Q(x) \]

\[ I(x) = e^{\int P(x)dx} \]

\[ yI(x) = \int I(x)Q(x)dx \]

WARNING: Do not forget your plus C! Also be sure to review the Applications of Differential Equations section.
7 Probability and Statistics

- $f(x)$ is a probability density function over $[a, b]$ if it is continuous and
  $f(x) \geq 0$ on $[a, b]$ and $\int_a^b f(x)dx = 1$

- Review the formulas for expected value, variance and standard deviation.

- The z-scores table should be given on the exam for you.

GOOD LUCK