MATH 220 U1 **Exam 3** **Fall 2014**

Name _____

- Do not open this exam packet until I say *start*.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- Remove hats and sunglasses.
- If you have a question, raise your hand and I will come to you. When you stand up, you are done with your exam.
- Quit working and close this packet when I say *stop*.
- Good luck!

POSSIBLE	[1] 15	[2] 16	[3] 16	[4] 12	[5] 15	[6] 8	[7] 8	[8] 15	[T] 100*
SCORE									

*It is possible to score a total of 105 points on this exam, but your score will be out of 100.

The following formulas might be useful.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

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1. (5 points each) Suppose that f is an even, continuous function and that

$$\int_0^2 f(x)\,dx = 5, \qquad \int_2^3 f(x)\,dx = 7.$$

Determine the following quantities. Please put $\boxed{\text{boxes}}$ around your answers.

(a) $\displaystyle\int_0^5 f(x)\,dx + \int_5^3 f(x)\,dx$

(b) $\displaystyle\int_{-2}^3 f(x)\,dx$

(c) $\displaystyle\int_0^2 f\left(\frac{x+4}{2}\right)\,dx$

2. (8 points each) Evaluate the following definite integrals. Please put boxes around your answers.

(a) $\displaystyle\int_0^{\pi/4} 5\sec^2 t\, dt$

(b) $\displaystyle\int_{-7}^{0} 7\sqrt{49 - x^2}\, dx$

3. (8 points each) Find the most general antiderivative. Please put boxes around your answers.

(a) $\int x\,(2-x)^{7/2}\,dx$

(b) $\int \frac{\arctan x}{1+x^2}\,dx$

4. (12 points) A particle moves along the x-axis with position $s(t)$, velocity $v(t)$, and acceleration $a(t)$. Given that

$$v(t) = \cos(\pi t) - \sin(\pi t),$$

and $s(0) = 0$ and $a(0) = -\pi$, find the particle's position, velocity, and acceleration at $t = 1/2$. (Don't worry about units.)

$s(1/2) =$

$v(1/2) =$

$a(1/2) =$

5. (a) (11 points) Evaluate the following sum/limit. Please put a box around your answer.

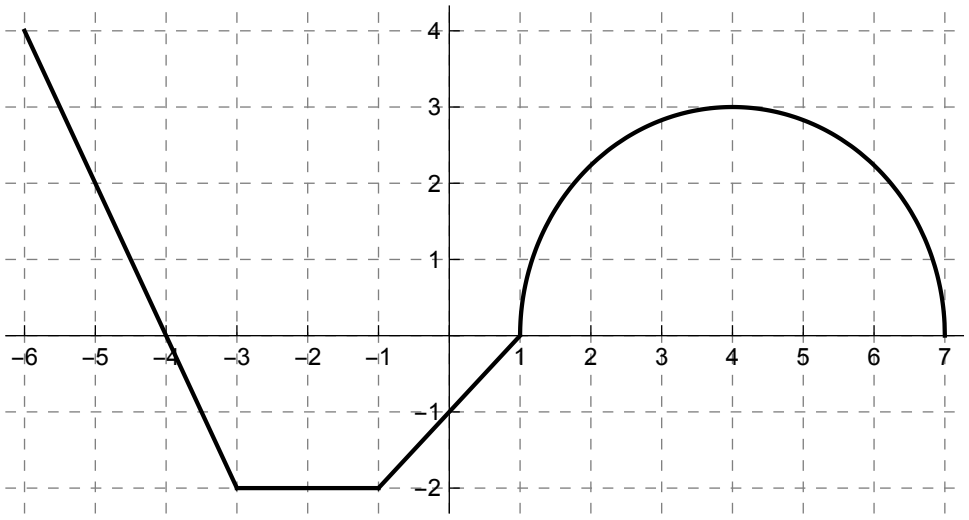
$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{4k}{n^2+2n} - \frac{12k^2}{n^3} + \frac{2}{5n^2} \right)$$

(b) (4 points) What is the sum of all the integers from 1 to 200?

$$1 + 2 + 3 + 4 + 5 + \ldots + 198 + 199 + 200 =$$

6. (8 points) Suppose that $f(4) = 6$ and that $f'(x) \leq 1$ for $-4 \leq x \leq 4$. Use the mean value theorem (for derivatives) to determine the smallest value that $f(-4)$ can be. Make sure to show sufficient work.

7. (8 points) The graph of $y = f(x)$ is shown below. You may assume that any curve that looks like a circle is, in fact, a circle.



What is the average value of $f(x)$ for $-4 \leq x \leq 4$? Please put a $\boxed{\text{box}}$ around your answer.

8. (15 points) Let $G(x) = \int_x^{x^2} \sqrt{8+t^4}\, dt$. Please put $\boxed{\text{boxes}}$ around your answers.

(a) Find $G'(x)$.

(b) Find the equation of the tangent line to $y = G(x)$ at $x = 1$.

(c) Use linear approximation to estimate

$$\int_{1.1}^{1.21} \sqrt{8+x^4}\, dx.$$