

16 September 2014    **Limits and Continuity**

(1) Evaluate the limit

$$\lim_{x \rightarrow 2} \arctan \left( \frac{x^2 - 4}{3x^2 - 6x} \right).$$

(2) Show that the function

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4, \\ \cos x & \text{if } x \geq \pi/4, \end{cases}$$

is continuous on  $(-\infty, \infty)$ .

(3) What value of  $f(2)$  makes the function

$$f(x) = \frac{x^3 - x^2 - 2x}{x - 2}$$

continuous on  $(-\infty, \infty)$ ?

(4) Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}.$$

- (5) Use the intermediate value theorem to prove the following statements.
- (a) The equation  $\sin x = x^3 - x$  has a solution in  $(1, 2)$ .

(b) If  $f(x) = x^2 + 10 \sin x$ , then there exists a number  $c$  such that  $f(c) = 1000$ .

- (6) Suppose that  $2x - 1 \leq f(x) \leq x^2$  for  $0 < x < 3$ . Evaluate

$$\lim_{x \rightarrow 1} f(x).$$

- (7) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{|2x - 1| - |2x + 1|}{x}.$$