## 13 October 2014 Graphing and Optimization

(1) Consider the function $f(x)=(x+1)^{5}-5 x-2$.
(a) Find all the critical $\left(f^{\prime}(x)=0\right)$ and crucial $\left(f^{\prime \prime}(x)=0\right)$ points of $f$.
(b) Fill in the chart below: first fill in $x$-coordinates and intervals. On the second and third lines, put a + or - in each box.

(c) Right below the table, make a rough sketch of $f$.
(d) Where is $f$ increasing/decreasing?
(e) Where are the local mins/maxes of $f$ ?
(f) Where is $f$ concave up/down?
(g) Where are the inflection points of $f$ ?
(2) Let $f(t)$ be the temperature at time $t$ in Urbana-Champaign, and suppose that at time $t=3$ you are shivering in the cold. How do you feel about the given data in each case?
(a) $f^{\prime}(3)=-2, \quad f^{\prime \prime}(3)=4$
(b) $f^{\prime}(3)=-2, \quad f^{\prime \prime}(3)=-4$
(c) $f^{\prime}(3)=2, \quad f^{\prime \prime}(3)=-4$
(d) $f^{\prime}(3)=2, \quad f^{\prime \prime}(3)=4$
(3) Suppose the derivative of a function $f$ is

$$
f^{\prime}(x)=(x-1)(x-2)^{2}(x-3)^{3}(x-4)^{4}(x-5)^{5}(x-6)^{6} .
$$

Make a chart like 1 (c), but only include rows for $x$ and $f^{\prime}(x)$. Where is $f$ increasing/decreasing?
(4) Consider the following problem: Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.
(a) Draw a picture; label the width $x$ and the height $y$.
(b) What are the smallest/largest values that $x$ and $y$ can be?
(c) Write an equation relating $x$ and $y$ to the perimeter of the rectangle.
(d) Find an expression for the area $A$ in terms of $x$ and $y$.
(e) Find a function $A(x)$ that gives the area in terms of just $x$.
(f) For what value of $x$ is $A$ maximized? (use the first or second derivative test to make sure you have a maximum)
(5) A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?
(6) If $1200 \mathrm{~cm}^{2}$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

