## 16 October 2014 L'Hospital's Rule

(1) Evaluating the following limits using L'Hospital's rule.

(a) 
$$\lim_{x \to 0} \frac{x^2}{1 - \cos x}$$

(b) 
$$\lim_{x \to 0} \frac{\ln \sqrt{x}}{x^2}$$

(c) 
$$\lim_{x \to 0} \frac{x}{\tan^{-1}(4x)}$$

(2) Sometimes you have to use an algebraic "trick" to put the indeterminate form in  $\frac{0}{0}$  form or  $\frac{\infty}{\infty}$  form. Evaluate the following limits by manipulating them until they look like fractions of the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ .

(a) 
$$\lim_{x \to 0} \left( \cot x - \frac{1}{x} \right)$$

(b) 
$$\lim_{x \to 1^+} (\ln(x^7 - 1) - \ln(x^5 - 1))$$

(3) When you encounter the indeterminate forms involving powers  $(1^{\infty}, 0^0, \infty^0)$  you should write the function as y = f(x) and then take the natural log of both sides. Now take the limit of  $\ln f(x)$  using L'Hospital's rule; when you finish, don't forget that you have found  $\ln y$ , not y.

(a)  $\lim_{x \to 0^+} (\tan 2x)^x$ .

(b)  $\lim_{x \to \infty} x^{\ln 2/(1 + \ln x)}$ 

(c) 
$$\lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^x$$