28 October 2014 **Mean Value Theorem / Antiderivatives**

(1) State the Mean Value Theorem.

(2) Verify the conclusion of the Mean Value Theorem (that is, find c) for the function

$$f(x) = \sqrt{1 + \cos x}$$

on the interval $[-\pi/2, \pi/2]$.

(3) Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x?

(4) For each function $f(x)$, find a function $F(x)$ such that $F'(x) = f(x)$.

(a) $f(x) = x^n$, $(n \neq -1)$

(b) $f(x) = 1/x$

(c) $f(x) = e^x$

(d) $f(x) = \cos x$

(e) $f(x) = \sin x$

(f) $f(x) = \sec^2 x$

(g) $f(x) = \sec x \tan x$

(h) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$

(i) $f(x) = \dfrac{1}{1+x^2}$

(5) For each problem below, find the **most general** function $f(x)$ that satisfies all the given conditions.

(a) $f'(x) = \dfrac{3}{1+x^2}, \quad f(\pi/4) = 7$

(b) $f'(x) = 3\dfrac{\sqrt{x}}{x^2} - \dfrac{12}{\sqrt{x}} + 14x^{7/2}$

(c) $f''(x) = 20x^3 + \sin x$

(d) $f'''(x) = 6x, \quad f''(1) = 3, \quad f'(0) = -4, \quad f(2) = 0$

(e) $f^{(173)}(x) = e^x, \quad f^{(n)}(0) = 1$ for all $n \geq 0$