29 October 2014 Antiderivatives and Differential Equations

(1) A particle is moving with acceleration

$$a(t) = 3\cos t - 2\sin t$$

and its position and velocity at time t = 0 are given by s(0) = 0 and v(0) = 4. Find s(t).

(2) Suppose you're driving a car and you slam on the brakes, decelerating at 16 ft/s², and your tires make 200 feet of skid marks before you come to a complete stop. How fast were you driving?

(3) You're in Canada driving 100 km/hr and you see a bunch of huge elk in front of you 80 meters away. You'd rather not total your car, so you slam on the brakes. What constant deceleration is required to stop in time?

(4) Radioactive substances decay at a rate proportional to the remaining mass. Adamantium-42 has a half-life of 1234 years (meaning that after 1234 years, only half of the Adamantium will remain). Suppose that today Wolverine has 100 g of Adamantium injected into his body. How much of the Adamantium will remain after t years? After how long will only 1 g remain?

(5) With continuous compounding of interest, the rate of increase of an investment is proportional to its size, where the proportionality constant is r, the interest rate. How long will it take an investment to double in value if the interest rate is 6% compounded continuously?

(6) Solve the following differential equations.

(a)
$$\frac{dy}{dx} = xy^2$$

(b)
$$\frac{dp}{dt} = t^2 p - p + t^2 - 1$$