The proof of Proposition 3.1 has a (patchable) hole arising from the fact that a nonunital ring homomorphism can kill each coefficient of a polynomial without killing the polynomial itself! The corrected proof proceeds as follows:

Proof. Assume that R is locally nilpotent and suppose that for a derivation δ the ring $A = R[x;\delta]$ is not Brown-McCoy. Then there exists a surjective ring homomorphism $\varphi : A \to S$ where S is a simple ring with 1. Let $f = a_0 + \cdots + a_n x^n \in A$ be such that $\varphi(f) = 1$. Consider the set $X := \{a_0, \ldots, a_n\} \subseteq R$. As R is locally nilpotent we have that X is nilpotent. Choose $m \geq 1$ such that $\varphi(X^m) = 0 \neq \varphi(X^{m-1})$ (where if $m = 1$, we consider the condition $\varphi(X^0) \neq 0$ vacuously fulfilled).

If $m > 1$ fix $r \in X^{m-1}$ with $\varphi(r) \neq 0$. We have

$$\varphi(r) = \varphi(r)\varphi(f)^2 = \varphi(rf^2) = \sum_{i=0}^{n} \varphi(rr_i x^i f) = \sum_{i=0}^{n} \varphi(rr_i)\varphi(x^i f) = 0$$

which is the needed contradiction.

If $m = 1$, we have

$$1 = \varphi(f)^2 = \varphi(f^2) = \sum_{i=0}^{n} \varphi(r_i x^i f) = \sum_{i=0}^{n} \varphi(r_i)\varphi(x^i f) = 0$$

which contradicts the fact that S is a simple (hence nonzero) ring. □