

MA 342 Pretest

1. Suppose $|f'(x)| \leq K$ for all $x \in \mathbb{R}$. Show that f is uniformly continuous on \mathbb{R} .
2. Show that $f(x) = 1/x$ is uniformly continuous on $[1, \infty)$.
3. Prove that if $\lim(a_n/n) = L > 0$ then $\lim(a_n) = +\infty$.
4. If $0 < r < 1$ and $|x_{n+1} - x_n| < r^n$ for all $n \in \mathbb{N}$, show that (x_n) is a Cauchy sequence.
5. Show that the series

$$\sum_{n=1}^{\infty} \frac{|\cos n|}{n^2}$$

is convergent.

6. If $f(x) := e^x$, show that the remainder term in Taylor's Theorem converges to zero as $n \rightarrow \infty$, for each fixed x_0 and x .
7. Show that if $f : A \rightarrow \mathbb{R}$ is continuous on $a \subseteq \mathbb{R}$ and if $n \in \mathbb{N}$, then the function f^n defined by $f^n(x) = (f(x))^n$ for $x \in A$, is continuous on A .
8. If f and g are continuous on $[a, b]$ and if $\int_a^b f = \int_a^b g$, prove that there exists $c \in [a, b]$ such that $f(c) = g(c)$.
9. Find $F'(x)$ when F is defined on $[0, 1]$ by: $F(x) := \int_0^{x^2} (1 + t^3)^{-1} dt$.
10. Determine the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{nx^n}{2^n}.$$

11. Evaluate the following integrals

(a) $\int_0^{\pi/2} (1 + 3 \sin \theta)^{3/2} \cos \theta \, d\theta$

(b) $\int x \sqrt{4 - x} \, dx$

(c) $\frac{2x + 1}{\sqrt{x^2 + x}} dx$

(d) $\int (\ln t)^2 dt$

(e) $\int x^2 \arctan x \, dx$

(f) $\int x^3 \cos x^2 \, dx$