

NS



001

Math 213
Practice Exam I
February 2020

Name: _____

Section: _____

Instructor: _____

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Instructions

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
- C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 3 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Please do not talk about the test with other students until after the last day of the exam.



Part I: Multiple Choice Questions: (3 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** mark in the other boxes.

1 Solve $3\mathbf{x} + 2\mathbf{u} = \mathbf{w}$ for \mathbf{x} if $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$.

$\mathbf{x} = \begin{bmatrix} 2/3 \\ -7/3 \\ 2 \end{bmatrix}$.

$\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$.

$\mathbf{x} = \begin{bmatrix} 4/3 \\ 2/3 \\ -1/3 \end{bmatrix}$.

$\mathbf{x} = \begin{bmatrix} 2/3 \\ 2 \\ -7/3 \end{bmatrix}$.

$\mathbf{x} = \begin{bmatrix} 2 \\ 2 \\ -7 \end{bmatrix}$.

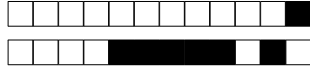
2 ♣ Which of the following statements is false? Mark all that apply.

For vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in \mathbb{R}^n if $\mathbf{u} + \mathbf{w} = \mathbf{v} + \mathbf{w}$ then $\mathbf{u} = \mathbf{v}$.

In \mathbb{R}^3 if two lines are *not* parallel then they must intersect in a point.

In \mathbb{R}^3 if two planes are *not* parallel then they must intersect in a line.

For vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in \mathbb{R}^n if $\mathbf{u} \cdot \mathbf{w} = \mathbf{v} \cdot \mathbf{w}$ then $\mathbf{u} = \mathbf{v}$.



3 ♣ Which of the following matrices is in row echelon form (REF).

$$\begin{bmatrix} 5 & 2 & -3 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 6 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

4 Which of the following statements is *false*?

$\|\mathbf{u} - \mathbf{v}\| \leq \|\mathbf{u}\| - \|\mathbf{v}\|$ for all vectors \mathbf{u} and \mathbf{v} .

If \mathbf{u} and \mathbf{v} are orthogonal then $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$.

$\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$ for all vectors \mathbf{u} and \mathbf{v} .

$\|c\mathbf{u}\| = |c|\|\mathbf{u}\|$ for all vectors \mathbf{u} and scalars c .

$|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\|\|\mathbf{v}\|$ for all vectors \mathbf{u} and \mathbf{v} .



5 ♣ Given that a system of equations has augmented matrix $[A|\mathbf{b}]$ that row reduces to the row echelon form $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$. Which of the following must be true about the system. Mark all that apply.

- The columns of A are linearly independent.
- The columns of A span all of \mathbb{R}^3
- The rank of A is 2.
- The system has 3 free variable.
- The system is inconsistent.
- The system has a unique solution.
- The system has infinitely many solutions.

6 Express the vector $\mathbf{b} = \begin{bmatrix} 5 \\ -7 \\ 8 \end{bmatrix}$ as a linear combination

$$\begin{bmatrix} 5 \\ -7 \\ 8 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

- $c_1 = -2, c_2 = 1$
- $c_1 = -2, c_2 = 3$
- $c_1 = 3, c_2 = -2$
- $c_1 = 2, c_2 = -3$
- $c_1 = 3, c_2 = 2$
- $c_1 = -1, c_2 = 2$
- \mathbf{b} is not a linear combination of these vectors



7 Find the projection of $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$.

$\begin{bmatrix} -1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$.

$\begin{bmatrix} -1/3 \\ -1/3 \\ -1/3 \end{bmatrix}$.

$\begin{bmatrix} 0 \\ -1/5 \\ -2/5 \end{bmatrix}$.

$\begin{bmatrix} 0 \\ 0 \\ 2/5 \end{bmatrix}$.

$\begin{bmatrix} 0 \\ -1/5 \\ 2/5 \end{bmatrix}$.

$\begin{bmatrix} 1/3 \\ -1/3 \\ -1/3 \end{bmatrix}$.

$\begin{bmatrix} -1/3 \\ 0 \\ -1/3 \end{bmatrix}$.

$\begin{bmatrix} -1/5 \\ 0 \\ 2/5 \end{bmatrix}$.



8 Let

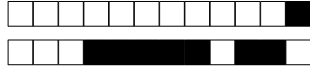
$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & s \\ 2 & 9 & 5 \end{bmatrix}.$$

What value(s) of s will make the matrix A noninvertible? Select one answer only.

- $s = 1$
- $s = 2$
- $s = 0$
- Any value except for $s = 1$.
- Any value except for $s = 0$.
- $s = 5$
- Any value except for $s = 2$.
- Any value except for $s = -1$.

9 ♣ Which of the following matrices are invertible? Mark all that apply.

- $\begin{bmatrix} 1 & -3 \\ 2 & -6 \end{bmatrix}$
- $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & 2 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
- $\begin{bmatrix} 0 & -4 & 0 \\ 2 & 0 & 8 \\ 0 & 7 & 0 \end{bmatrix}$



10 ♣ Which of the following sets of vectors spans all of \mathbb{R}^n for the appropriate n ?

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} \right\}$

11 ♣ Let $\mathbf{v}_1, \dots, \mathbf{v}_k$ be vectors in \mathbb{R}^n , and let A be the matrix whose columns are $\mathbf{v}_1, \dots, \mathbf{v}_k$. Which of the following is always true? Mark all that apply

If $\text{rank}(A) = k$, then $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly independent.

If $\mathbf{v}_1 = 2\mathbf{v}_2$ then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly dependent set.

If $k < n$, then $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly independent.

If $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly independent, they must span all of \mathbb{R}^n .

If $k > n$, then $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly dependent.

If $A\mathbf{x} = \mathbf{0}$ is consistent, then $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly independent.



12 Find the inverse of the elementary matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 0 1 2 3 4 5 6 7 8 9 10 **DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

a) True or False: A homogeneous system always has infinitely many solutions.

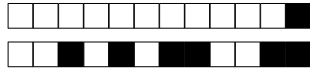
b) True or False: The vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ span all of \mathbb{R}^2 .

c) If $B = \begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix}$, then $\det B =$ _____.

d) Given that $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ -3 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$, $\mathbf{z} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, calculate

$$\frac{(\mathbf{x} + \mathbf{y}) \cdot \mathbf{z}}{\mathbf{x} \cdot (\mathbf{y} + \mathbf{z})}$$

e) Find the equation for the plane passing through the point $P = (1, 0, 2)$ and with normal vector $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.



14

0 1 2 3 4 5 6 7 8 9 10 **DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

a) True or false: If A, B and C are 3×3 matrices and $AB = AC$, then $B = C$.

b) True or false: The matrices $\begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ commute. _____

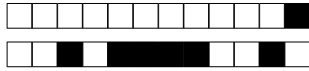
c) True or false: If A is a 3×4 matrix, then AA^T is symmetric. _____

d) How many solutions are there to a system of linear equations with augmented matrix that row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & 3 & -4 \end{array} \right]$? _____

e) Write the system

$$\begin{aligned} 2x - 3y &= -1 \\ -5x + 2y &= 3 \end{aligned}$$

as a vector equation (a linear combination of vectors equal to some vector).

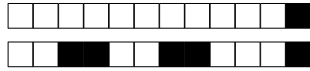


Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

15

0 1 2 3 4 5 6 7 8 **DON'T MARK**

Geometrically show what is meant by adding the two vectors $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.



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16

0 1 2 3 4 5 6 7 8 **DON'T MARK**

Solve the system of equations. If the system is inconsistent, state so. If there are infinitely many solutions, express them in parametric-vector form.

$$\begin{aligned}x - 2y + 3z &= -1 \\-2x + 3y + z &= 0 \\-x + y + 4z &= -1\end{aligned}$$



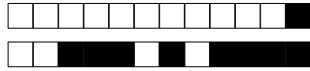
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17

0 1 2 3 4 5 6 7 8 **DON'T MARK**

Find the inverse of the matrix

$$A = \begin{bmatrix} -2 & -1 & 0 & 0 \\ -3 & -1 & 1 & 1 \\ -6 & -2 & 1 & 1 \\ 6 & 2 & -1 & 0 \end{bmatrix}.$$



18

0 1 2 3 4 5 6 7 8 **DON'T MARK**

Consider the matrices

$$A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \text{and} \quad A_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Is the matrix $B = \begin{bmatrix} 0 & 4 \\ 7 & 9 \end{bmatrix}$ in $\text{Span}\{A_1, A_2, A_3\}$? Justify your answer.