NS 40125367 001

Math 213
Practice Exam II
March 23–25, 2020

Name:____________________

Section:____________________

Instructor:____________________

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Instructions

A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
D) Multiple choice questions are worth 4 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
F) No books, notes, or calculators are allowed.
G) Please do not talk about the test with other students until after the last day of the exam.

Part I: Multiple Choice Questions: *(4 points each) Questions marked with a ♣ may have more than one correct answer. Mark* ***all*** *correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but DO NOT mark in the other boxes.*

1 ♣ Suppose that $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation. Which of the following statements about T must be true? Mark all that apply.

- ☐ $T(\mathbf{x}) = A\mathbf{x}$ for some $m \times n$ matrix A.
- ☐ The codomain of T is $\mathbb{R}^n$.
- ☐ For every vector $\mathbf{b}$ in $\mathbb{R}^m$, there is an $\mathbf{x}$ in $\mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{b}$.
- ☐ $T(c\mathbf{u} + d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$ for all $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, and all c and d in $\mathbb{R}$.
- ☐ $T(\mathbf{0}) = \mathbf{0}$.
- ☐ The range of T is a subspace of $\mathbb{R}^m$.

2 ♣ Which of the following sets are subspaces of $\mathbb{R}^2$? Mark all that apply.

- ☐ $\mathbb{R}^2$
- ☐ Span $\left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- ☐ The set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ with $x = y^2$.
- ☐ Row A, where A is a 4×2 matrix.
- ☐ The line in $\mathbb{R}^2$ defined by the equation $x - y + 4 = 0$.

3 Let $\mathcal{B}$ be the basis for $\mathbb{R}^3$ given by

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}.$$

Find the coordinator vector of $\mathbf{x} = \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}$ with respect to the basis $\mathcal{B}$.

- ☐ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.
- ☐ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}$.
- ☐ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$.
- ☐ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}$.
- ☐ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}$.
- ☐ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

4 Let S be the set consisting of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ in $\mathbb{R}^2$ with $xy \geq 0$. Consider the following statements:

I. S contains the zero vector.

II. S is closed under vector addition.

III. S is closed under scalar mulitplication.

Which of the following statements is true?

- ☐ Statements I., II., and III. are true, and hence S is a subspace.
- ☐ Statements II. and III. are true, while I. is false.
- ☐ Statements I. and III. are true, while II. is false.
- ☐ Statements I. and II. are true, while III. is false.
- ☐ Statement I., II., and III. are all false.
- ☐ Statement II. is true, while I. and III. are false.
- ☐ Statement I. is true, while II. and III. are false.
- ☐ Statement III. is true, while I. and II. are false.

5 ♣ Let A be a 5×7 matrix. Which of the following statements must be true? Mark all that apply.

- ☐ The columns of A span $\mathbb{R}^5$.
- ☐ $\dim(\operatorname{Col} A) + \dim(\operatorname{Nul} A) = 5$
- ☐ $\operatorname{rank} A$ is either 5, 6, or 7.
- ☐ $\dim(\operatorname{Col} A) = \dim(\operatorname{Row} A)$
- ☐ Nullity $A \geq 2$
- ☐ The columns of A are linearly dependent.

6 ♣ Which of the following sets give a basis for $\mathbb{R}^n$ (for the appropriate n)? Mark all that apply.

- ☐ $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 7 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

7 ♣ Let A be an $n \times n$ matrix. Which of the following must be true? Mark all that apply.

- ☐ If $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for some $\mathbf{b} \in \mathbb{R}^{\mathbf{n}}$, then $\det(A) \neq 0$.
- ☐ If $\det(A) \neq 0$, then the columns of A are linearly independent.
- ☐ If $row(A) \neq \mathbb{R}^n$, $\det(A) \neq 0$.
- ☐ If $Nul(A) = \{\mathbf{0}\}$, then $\det(A) \neq 0$.
- ☐ If $\det(A) = 0$, $rank(A) < n$.
- ☐ If $\det(A) = 0$, the columns of A span all of $\mathbb{R}^n$.

8 If A and B are $n \times n$ matrices with $\det(A) = -3$ and $\det(B) = 4$, what is $\det(A^2 B^{-1}(A^{-1}B^2)^T)$?

- ☐ 0
- ☐ 1
- ☐ -3
- ☐ 4
- ☐ -12
- ☐ Not enough information to determine.
- ☐ 27/4
- ☐ 12

9 ♣ Let $\mathbf{x}$ be an eigenvector of the $n \times n$ matrix A, with corresponding eigenvalue λ. Let I_n denote the $n \times n$ identity matrix. Which of the following must be true? Mark all that apply.

- ☐ $A^2\mathbf{x} = 2\lambda\mathbf{x}$.
- ☐ If A is invertible, $A^{-1}\mathbf{x} = \lambda\mathbf{x}$.
- ☐ $rank(A - \lambda I_n) = n$
- ☐ $(A - \lambda I)\mathbf{x} = \mathbf{0}$.
- ☐ If c is some nonzero scalar, $c\mathbf{x}$ is also an eigenvector of A with eigenvalue λ.
- ☐ $A\mathbf{x} = \lambda\mathbf{x}$

10 ♣ Suppose A is similar to B. Which of the following must be true? Mark all that apply.

- ☐ A^T is similar to B^T.
- ☐ $\det(A) = \dfrac{1}{\det(B)}$
- ☐ A and B have the same eigenvalues.
- ☐ $A + B$ is similar to A.
- ☐ A^{-1} is similar to B^{-1}.
- ☐ If C is also similar to A, then C is similar to B too.

11 ♣ Suppose A is a 6×6 matrix with four distinct eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. Which of the following would guarantee that A is diagonalizable?

- ☐ λ_1 has algebraic multiplicity 3.
- ☐ λ_1 has geometric multiplicity 3.
- ☐ A has 6 linearly independent eigenvectors.
- ☐ Each eigenvalue has geometric multiplicity 1.
- ☐ λ_1 and λ_2 both have algebraic multiplicity 2.
- ☐ λ_1 and λ_2 both have geometric multiplicity 2.

Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

12 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 **DON'T MARK**

a) State the precise definition of a subspace S of $\mathbb{R}^n$:

b) State the precise definition of a basis $\mathcal{B}$ for a subspace S:

c) Let $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ be a basis for $\mathbb{R}^2$, and let $\mathbf{x} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$. Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of $\mathbf{x}$ with respect to the basis $\mathcal{B}$.

d) True or False: If S is a 4-dimensional subspace of $\mathbb{R}^8$, and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4, \mathbf{b}_5\}$ is a set of vectors that spans S, then $\mathcal{B}$ is a basis for S.

e) True or False: If $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors that spans a subspace H, then $\{\mathbf{v}_1, \mathbf{v}_2 - \mathbf{v}_1, \mathbf{v}_3 - \mathbf{v}_1\}$ will be a basis for H.

f) Find a basis for the subspace of $\mathbb{R}^4$ given by

$$W = \operatorname{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 2 \\ 1 \end{bmatrix} \right\},$$

and find $\dim W =$ ____________.

g) True of False: $\det(AB) = \det(BA)$

h) If A is an 3×3 matrix with $\det A = 5$, then $\det(2A) =$__________.

Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 **DON'T MARK**

Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & -3 \\ -3 & 1 & 0 & -5 & -5 & 10 \\ -1 & 2 & 1 & -4 & -7 & 8 \\ -4 & -2 & 3 & 3 & -6 & 19 \end{bmatrix}.$$

Find bases for Row A, Col A, and Null A. Clearly label which basis is which.

14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 **DON'T MARK**

Let $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ be linear transformations defined by

$$T(\mathbf{x}) = \operatorname{proj}_{\mathbf{u}} \mathbf{x} \qquad \text{and} \qquad S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ x + y \end{bmatrix}.$$

Find the standard matrix for the linear transformation $S \circ T(\mathbf{x}) = S(T(\mathbf{x}))$.

15 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 **DON'T MARK**

Diagonalize the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

In other words, find a diagonal matrix D and an invertible matrix P so that $A = PDP^{-1}$.

16 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 **DON'T MARK**

Compute the determinant of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 3 & 2 & -1 & 1 \\ 4 & -2 & 3 & 2 \end{bmatrix}$.