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**Math 213**  
**Practice Exam II**  
March 23–25, 2020

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Instructor: \_\_\_\_\_

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**Instructions**

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
- C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 4 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Please do not talk about the test with other students until after the last day of the exam.



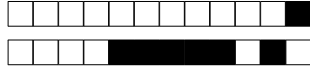
**Part I: Multiple Choice Questions:** (4 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** mark in the other boxes.

1 ♣ Suppose that  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a linear transformation. Which of the following statements about  $T$  must be true? Mark all that apply.

- $T(\mathbf{x}) = A\mathbf{x}$  for some  $m \times n$  matrix  $A$ .
- The codomain of  $T$  is  $\mathbb{R}^n$ .
- For every vector  $\mathbf{b}$  in  $\mathbb{R}^m$ , there is an  $\mathbf{x}$  in  $\mathbb{R}^n$  such that  $T(\mathbf{x}) = \mathbf{b}$ .
- $T(c\mathbf{u} + d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$  for all  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbb{R}^n$ , and all  $c$  and  $d$  in  $\mathbb{R}$ .
- $T(\mathbf{0}) = \mathbf{0}$ .
- The range of  $T$  is a subspace of  $\mathbb{R}^m$ .

2 ♣ Which of the following sets are subspaces of  $\mathbb{R}^2$ ? Mark all that apply.

- $\mathbb{R}^2$
- $\text{Span} \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- The set of all vectors  $\begin{bmatrix} x \\ y \end{bmatrix}$  with  $x = y^2$ .
- Row  $A$ , where  $A$  is a  $4 \times 2$  matrix.
- The line in  $\mathbb{R}^2$  defined by the equation  $x - y + 4 = 0$ .



3 Let  $\mathcal{B}$  be the basis for  $\mathbb{R}^3$  given by

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}.$$

Find the coordinator vector of  $\mathbf{x} = \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}$  with respect to the basis  $\mathcal{B}$ .

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}.$

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}.$

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix}.$

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 3 \\ -5 \end{bmatrix}.$

$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}.$



4 Let  $S$  be the set consisting of all vectors  $\begin{bmatrix} x \\ y \end{bmatrix}$  in  $\mathbb{R}^2$  with  $xy \geq 0$ . Consider the following statements:

- I.  $S$  contains the zero vector.
- II.  $S$  is closed under vector addition.
- III.  $S$  is closed under scalar multiplication.

Which of the following statements is true?

- Statements I., II., and III. are true, and hence  $S$  is a subspace.
- Statements II. and III. are true, while I. is false.
- Statements I. and III. are true, while II. is false.
- Statements I. and II. are true, while III. is false.
- Statement I., II., and III. are all false.
- Statement II. is true, while I. and III. are false.
- Statement I. is true, while II. and III. are false.
- Statement III. is true, while I. and II. are false.

5 ♣ Let  $A$  be a  $5 \times 7$  matrix. Which of the following statements must be true? Mark all that apply.

- The columns of  $A$  span  $\mathbb{R}^5$ .
- $\dim(\text{Col } A) + \dim(\text{Nul } A) = 5$
- $\text{rank } A$  is either 5, 6, or 7.
- $\dim(\text{Col } A) = \dim(\text{Row } A)$
- $\text{Nullity } A \geq 2$
- The columns of  $A$  are linearly dependent.



6 ♣ Which of the following sets give a basis for  $\mathbb{R}^n$  (for the appropriate  $n$ )? Mark all that apply.

$\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 7 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

7 ♣ Let  $A$  be an  $n \times n$  matrix. Which of the following must be true? Mark all that apply.

If  $A\mathbf{x} = \mathbf{b}$  has infinitely many solutions for some  $\mathbf{b} \in \mathbb{R}^n$ , then  $\det(A) \neq 0$ .

If  $\det(A) \neq 0$ , then the columns of  $A$  are linearly independent.

If  $\text{row}(A) \neq \mathbb{R}^n$ ,  $\det(A) \neq 0$ .

If  $\text{Nul}(A) = \{\mathbf{0}\}$ , then  $\det(A) \neq 0$ .

If  $\det(A) = 0$ ,  $\text{rank}(A) < n$ .

If  $\det(A) = 0$ , the columns of  $A$  span all of  $\mathbb{R}^n$ .

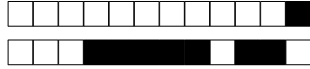


8 If  $A$  and  $B$  are  $n \times n$  matrices with  $\det(A) = -3$  and  $\det(B) = 4$ , what is  $\det(A^2B^{-1}(A^{-1}B^2)^T)$ ?

- 0
- 1
- 3
- 4
- 12
- Not enough information to determine.
- 27/4
- 12

9 ♣ Let  $\mathbf{x}$  be an eigenvector of the  $n \times n$  matrix  $A$ , with corresponding eigenvalue  $\lambda$ . Let  $I_n$  denote the  $n \times n$  identity matrix. Which of the following must be true? Mark all that apply.

- $A^2\mathbf{x} = 2\lambda\mathbf{x}$ .
- If  $A$  is invertible,  $A^{-1}\mathbf{x} = \lambda\mathbf{x}$ .
- $\text{rank}(A - \lambda I_n) = n$
- $(A - \lambda I)\mathbf{x} = \mathbf{0}$ .
- If  $c$  is some nonzero scalar,  $c\mathbf{x}$  is also an eigenvector of  $A$  with eigenvalue  $\lambda$ .
- $A\mathbf{x} = \lambda\mathbf{x}$



10 ♣ Suppose  $A$  is similar to  $B$ . Which of the following must be true? Mark all that apply.

- $A^T$  is similar to  $B^T$ .
- $\det(A) = \frac{1}{\det(B)}$
- $A$  and  $B$  have the same eigenvalues.
- $A + B$  is similar to  $A$ .
- $A^{-1}$  is similar to  $B^{-1}$ .
- If  $C$  is also similar to  $A$ , then  $C$  is similar to  $B$  too.

11 ♣ Suppose  $A$  is a  $6 \times 6$  matrix with four distinct eigenvalues  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ . Which of the following would guarantee that  $A$  is diagonalizable?

- $\lambda_1$  has algebraic multiplicity 3.
- $\lambda_1$  has geometric multiplicity 3.
- $A$  has 6 linearly independent eigenvectors.
- Each eigenvalue has geometric multiplicity 1.
- $\lambda_1$  and  $\lambda_2$  both have algebraic multiplicity 2.
- $\lambda_1$  and  $\lambda_2$  both have geometric multiplicity 2.



**Part II: Short Answer Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

12 0 1 2 3 4 5 6 7 8 9 10 **DON'T MARK**

- a) State the precise definition of a subspace  $S$  of  $\mathbb{R}^n$ :
- b) State the precise definition of a basis  $\mathcal{B}$  for a subspace  $S$ :
- c) Let  $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$  be a basis for  $\mathbb{R}^2$ , and let  $\mathbf{x} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ . Find the coordinate vector  $[\mathbf{x}]_{\mathcal{B}}$  of  $\mathbf{x}$  with respect to the basis  $\mathcal{B}$ .
- d) True or False: If  $S$  is a 4-dimensional subspace of  $\mathbb{R}^8$ , and  $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4, \mathbf{b}_5\}$  is a set of vectors that spans  $S$ , then  $\mathcal{B}$  is a basis for  $S$ .
- e) True or False: If  $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is a linearly independent set of vectors that spans a subspace  $H$ , then  $\{\mathbf{v}_1, \mathbf{v}_2 - \mathbf{v}_1, \mathbf{v}_3 - \mathbf{v}_1\}$  will be a basis for  $H$ .
- f) Find a basis for the subspace of  $\mathbb{R}^4$  given by

$$W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 2 \\ 1 \end{bmatrix} \right\},$$

and find  $\dim W =$  \_\_\_\_\_.

- g) True or False:  $\det(AB) = \det(BA)$
- h) If  $A$  is an  $3 \times 3$  matrix with  $\det A = 5$ , then  $\det(2A) =$ \_\_\_\_\_.





**Part III: Free Response Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

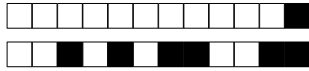
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0 1 2 3 4 5 6 **DON'T MARK**

Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & -3 \\ -3 & 1 & 0 & -5 & -5 & 10 \\ -1 & 2 & 1 & -4 & -7 & 8 \\ -4 & -2 & 3 & 3 & -6 & 19 \end{bmatrix}.$$

Find bases for Row  $A$ , Col  $A$ , and Null  $A$ . Clearly label which basis is which.



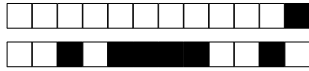
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0 1 2 3 4 5 6 **DON'T MARK**

Let  $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ , and let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  and  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be linear transformations defined by

$$T(\mathbf{x}) = \text{proj}_{\mathbf{u}} \mathbf{x} \quad \text{and} \quad S \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x - y \\ x + y \end{bmatrix}.$$

Find the standard matrix for the linear transformation  $S \circ T(\mathbf{x}) = S(T(\mathbf{x}))$ .



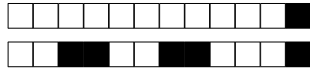
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0 1 2 3 4 5 6 7 **DON'T MARK**

Diagonalize the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

In other words, find a diagonal matrix  $D$  and an invertible matrix  $P$  so that  $A = PDP^{-1}$ .



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16

0 1 2 3 4 5 6 7 **DON'T MARK**

Compute the determinant of the matrix  $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 3 & 2 & -1 & 1 \\ 4 & -2 & 3 & 2 \end{bmatrix}$ .