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001

**Math 213**  
**Practice Exam I**  
October 2019

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Instructor: \_\_\_\_\_

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**Instructions**

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
- C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 3 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Please do not talk about the test with other students until after the last day of the exam.



**Part I: Multiple Choice Questions:** (3 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** mark in the other boxes.

1 ♣ Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Which of the following matrices are row equivalent to  $A$ ? Mark all that apply.

$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 5 & 0 & 3 \end{bmatrix}$

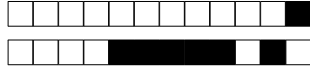
$\begin{bmatrix} 2 & 0 & 4 & 0 & 2 \\ 0 & 2 & 2 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 1 & 1 & 3 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 & 1 \end{bmatrix}$



2 ♣ Which of the following augmented matrices will correspond to systems of linear equations with a unique solution? Mark all that apply.

$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 4 \end{array} \right]$

$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & 7 \end{array} \right]$

$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 7 \\ 3 & 6 & 9 & 12 \end{array} \right]$

$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right]$

$\left[ \begin{array}{cc|c} 1 & 2 & 2 \\ 1 & 1 & -3 \end{array} \right]$



3 ♣ Which of the following sets of vectors are linearly independent? Mark all that apply.

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$



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4 ♣ Which of the following vectors is in

$$\text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}?$$

Mark all that apply.

$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -3 \\ 2 \\ -3 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$



5 ♣ Suppose that  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \in \mathbb{R}^4$ , and that the set of vectors

$$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$$

is linearly independent. Which of the following *must* be true? Mark all that apply.

- One of the vectors  $\mathbf{v}_j$  can be expressed as a linear combination of the other vectors.
- The matrix  $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4]$  has a pivot in every column.
- For any vector  $\mathbf{b} \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ , the equation  $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4 = \mathbf{b}$  has infinitely many solutions.
- The rank of the matrix  $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4]$  is 4.
- The set  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is linearly independent.
- The system of equations corresponding to the augmented matrix  $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4 \mid \mathbf{b}]$  is consistent for all choices of  $\mathbf{b} \in \mathbb{R}^4$ .

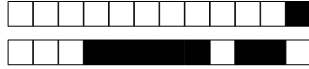
6 ♣ The points  $A = (-1, -3, 5), B = (12, 4, 6), C = (16, 18, 22), D = (1, 2, 0) \in \mathbb{R}^3$  form the corners of a four-sided figure whose sides give the vectors  $\vec{AB}, \vec{BC}, \vec{CD}, \vec{DA}$ . At which of the points  $A, B, C, D$  are these vectors orthogonal?

- B
- C
- none of them
- A
- D

7 Let  $v_1 = [1, 2, -3], v_2 = [2, 3, -5], v_3 = [6, -4, -1], v_4 = [1, 2, -1] \in \mathbb{R}^3$  be vectors. Let  $v = [x, y, z]$ . Solve the vector equation

$$-2v_1 + 4v_2 + v = 7v_3 + 2v_4.$$

- $[38, 32, -5]$
- $[-38, -32, 5]$
- $[38, -32, 5]$
- $[-38, 32, 5]$
- $[-38, -32, -5]$



8 Let  $v = [1, 1]$ ,  $w = [1, 0]$  and find the angle between  $v$  and  $w$

- $\pi/2$
- $2\pi/3$
- $0$
- $\pi/4$
- $\pi/6$
- $\pi/3$

9 Let  $A$ ,  $B$ , and  $C$  be  $n \times n$  matrices and  $r$  a scalar. Which of the following properties of matrix transpose does *not* always hold.

- $(AB)^T = A^T B^T$
- $(rA)^T = rA^T$
- $(A + B)^T = A^T + B^T$
- $(A^T)^{-1} = (A^{-1})^T$  (assuming  $A$  is invertible)
- $(A^T)^T = A$

10 ♣ Which of the following statements are always true? Mark all that apply.

- If  $A$  is  $n \times n$  and invertible, then  $A\mathbf{x} = \mathbf{b}$  has a unique solution for every  $\mathbf{b} \in \mathbb{R}^n$ .
- If  $A$  is  $n \times n$  and invertible, then its columns span  $\mathbb{R}^n$ .
- If  $A$  is  $m \times n$  and  $n < m$ , then the columns of  $A$  are linearly independent.
- If  $A$  is  $m \times n$  and  $n > m$ , then the columns of  $A$  are linearly dependent.
- If  $A$  is  $m \times n$  and  $n > m$ , then the columns of  $A$  span  $\mathbb{R}^m$ .
- If  $A$  is  $m \times n$  and  $\text{rank}(A) = n$ , then  $A$  is invertible.

11 ♣ Given that  $A$  and  $B$  are invertible, which of the following statements are always true? Mark all that apply.

- $(-A)^{-1} = -(A^{-1})$
- $AA^{-1} = A^{-1}A$
- $(AB)^{-1} = A^{-1}B^{-1}$
- $(A^{-1})^{-1} = A^{-1}$
- $(A + B)^{-1} = A^{-1} + B^{-1}$
- None of these is true.



12 Find the inverse of the following elementary matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$





**Part II: Short Answer Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 0 1 2 3 4 5 6 7 8 9 10  
11 12

**DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

- a) Find the projection of the point  $(1, -1)$  onto the line with equation  $y = 2x$ .

\_\_\_\_\_

- b) The vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} 7 \\ -2 \\ -5 \\ 2 \end{bmatrix}$$

form a linearly dependent set. Write a dependence relation for these vectors:

\_\_\_\_\_.

- c) Define the span of a set of vectors  $\mathbf{v}_1, \dots, \mathbf{v}_p$ : \_\_\_\_\_

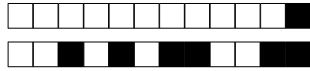
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- d) Circle the pivot positions in the matrix

$$\begin{bmatrix} -1 & 1 & -1 & 1 & 2 \\ 0 & 0 & 5 & 0 & 5 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

- e) If  $A$  is  $m \times n$  and  $B$  is  $n \times k$  then  $AB$  is \_\_\_\_\_  $\times$  \_\_\_\_\_.

- f) Is the matrix  $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$  invertible?



14

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<input type="checkbox"/>	11	<input type="checkbox"/>	12																		

**DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

- a) Let  $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$  and  $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 4 \end{bmatrix}$ . If  $\mathbf{w} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ k \end{bmatrix}$ , find the value of  $k$  so that  $\{\mathbf{v}, \mathbf{u}, \mathbf{w}\}$  forms a linearly dependent set.

$$k = \underline{\hspace{2cm}}$$

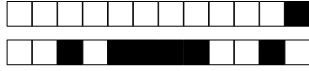
- b) True or False: If  $A^2 = I$  then  $A$  is invertible.
- c) True or False: If  $A$  is  $m \times n$  and  $B$  is  $k \times m$ , then  $BA$  is undefined.
- d) Write the following system as a matrix equation  $A\mathbf{x} = \mathbf{b}$ :

$$2x + 3y = 7, \quad -3y + 5x = -2$$

e)  $(A^n)^{-1} = \underline{\hspace{2cm}}$

- f) Compute the matrix product

$$\begin{bmatrix} 1 & 2 \\ -2 & 3 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & -6 \\ -1 & 3 & 5 \end{bmatrix} = \underline{\hspace{2cm}}$$

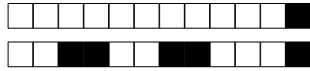


**Part III: Free Response Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

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0 1 2 3 4 5 6 7 **DON'T MARK**

Describe all vectors that are orthogonal to  $(-7, 10) \in \mathbb{R}^2$ .

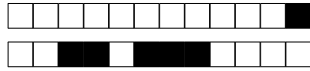


16

0 1 2 3 4 5 6 7 **DON'T MARK**

Let  $\vec{u}, \vec{v} \in \mathbb{R}^n$  be non-zero vectors. Explain why

$$\text{proj}_{\vec{u}}(\vec{v}) = \text{proj}_{\vec{u}}(\text{proj}_{\vec{u}}(\vec{v})).$$



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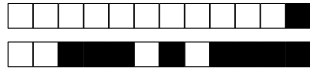
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0 1 2 3 4 5 6 7 **DON'T MARK**

Consider the planes in  $\mathbb{R}^3$  defined by the equations

$$2x + 2y + 6z = 14 \quad \text{and} \quad -3x - 2y - 7z = -16.$$

Find the *vector form* equation of the line of intersection between these two planes.



18

0 1 2 3 4 5 6 7 **DON'T MARK**

Determine if the matrix  $A$  is invertible, and if so find  $A^{-1}$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 3 & 0 & -1 \end{bmatrix}.$$