Final Exam (16537454)

Due: Wed, Apr 22, 2020 11:59 PM MDT

Question	1	2	3	4	5	6	7	8	9	10	11

Instructions

This exam is closed book. You are not allowed to use any notes, books, calculators, or other resources (online, print, or in-person). You are not allowed to speak to anyone about the contents of the exam until after the exam grades have been returned. You have 5 hours to complete the exam. Read all questions and instructions carefully. In some questions you must input your answers in a specific format which will be included in the instructions. Failure to follow instructions may result in losing points.

---

1. Question Details Identifying information - final exam [4646694] -

Please enter your NetID in all lowercase letters: ______

Please enter your BYU ID number with no spaces or dashes: ______

This exam is closed book. You are not allowed to use any notes, books, calculators, or other resources (online, print, or in-person). You are not allowed to speak to anyone about the contents of the exam until after the exam grades have been returned.

- [ ] I will not use any notes, books, calculators, or other resources (online, print, or in-person). I will not speak to anyone about the contents of the exam until after the exam grades have been returned.

You have 5 hours to complete the exam.

Read all questions and instructions carefully. In some questions you must input your answers in a specific format which will be included in the instructions. Failure to follow instructions may result in losing points.

2. Question Details Problems 1-12 [4645970]

Problems 1-12

1. Let A be a 3× 4 matrix corresponding to the coefficient matrix of a system with variables x_1, x_2, x_3, x_4 whose reduced row echelon form (RREF) is

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which of the following must be true? Mark all that apply.

- $rank(A)=2$
- A has a non-trivial null space.
- Variables x_3 and x_4 are free.
- If $A\mathbf{x}=\mathbf{b}$ is consistent, it has infinitely many solutions.
- The columns of A span $\mathbf{R}^3$.
- $dim(row(A))=4$

2. Let $T:\mathbf{R}^2\to\mathbf{R}^2$ be the linear transformation that reflects $\mathbf{x}$ over the line $y=-x$. What is the standard matrix $[T]$ for this linear transformation?

- $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

3. Let A,B,C,X be $n\times n$ invertible matrices. If $(2AX^{-1}B^{-1})^{-1}=C$, solve for the matrix X.

- $X=\frac{1}{2}B^{-1}CA$
- $X=2B^{-1}AC$
- $X=\frac{1}{2}B^{-1}AC$
- $X=2B^{-1}CA$
- $X=2A^{-1}CB$
- $X=\frac{1}{2}ACB^{-1}$
- $X=\frac{1}{2}ACB$
- $X=2AB^{-1}C$

4. Let A be an $n\times n$ invertible matrix. Which of the following must be true? Mark all that apply.

- $det(A)=0$
- The columns of A are linearly independent.
- $nullity(A)=0$
- $row(A)=\boldsymbol{R}^n$
- $rank(A)=n$
- $A\boldsymbol{x}=\boldsymbol{b}$ has infinitely many solutions for all $\boldsymbol{b}$ in $\boldsymbol{R}^n$.
- The reduced row echelon form of A has a row of all zeroes.
- A^T is also invertible.

5. If $det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}=6$ what is $det\begin{bmatrix} b & 2a & c \\ e & 2d & f \\ h & 2g & i \end{bmatrix}$?

- 12
- -12
- 6
- -6
- 2
- -2
- 0

6. Let A be a ***symmetric*** 4×4 matrix with eigenvalues 0,1,1,3 and corresponding eigenvectors $\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4$. Which of the following must be true? Mark all that apply.

- [ ] $A^T = A^{-1}$
- [ ] $A^T = A$
- [ ] $rank(A) = 3$
- [ ] $\mathbf{x}_1 \cdot \mathbf{x}_2 = 0$
- [ ] A is invertible.
- [ ] A is orthogonally diagonalizable.
- [ ] Eigenvalue $\lambda = 1$ has geometric multiplicity 2.
- [ ] $\mathbf{x}_1 - \mathbf{x}_2 = \mathbf{0}$

7. Compute the characteristic polynomial of the matrix $\begin{bmatrix} -5 & 0 & 4 \\ 0 & 3 & 0 \\ -8 & 0 & 7 \end{bmatrix}$.

- $-\lambda^3 + 5\lambda^2 - 3\lambda - 9$
- $-\lambda^3 + 5\lambda^2 - 6\lambda + 2$
- $-\lambda^3 + 5\lambda^2 - 6\lambda$
- $-\lambda^3 + 5\lambda^2 - 7\lambda + 3$
- $-\lambda^3 + 5\lambda^2 - 6\lambda - 8$
- $-\lambda^3 + 5\lambda^2 + 6\lambda$
- None of the above.

8. The matrix $\begin{bmatrix} -5 & 0 & 4 \\ 0 & 3 & 0 \\ -8 & 0 & 7 \end{bmatrix}$ (the same matrix as in the problem above) is similar to which of the following diagonal matrices?

- $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 6 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
- $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- This matrix is not diagonalizable.
- The matrix is diagonalizable, but not similar to any of the above.

9. If A is an $n\times n$ matrix with eigenvalue $\lambda=3$ and corresponding eigenvector $\boldsymbol{x}$, which of the following is true? Mark all that apply.

- $2\boldsymbol{x}$ is an eigenvector of A with eigenvalue 6.
- 9 is an eigenvalue of A^2.
- If A is invertible, $A^{-1}\boldsymbol{x}=\frac{1}{3}\boldsymbol{x}$.
- $(A-3I)\boldsymbol{x}=0$
- $\boldsymbol{x}=0$
- $rank(A-3I)=n$

10. Let A be a 3×5 matrix with rank 3. Which of the following statements must be true? Mark all that apply.

- The equation $A\boldsymbol{x} = \mathbf{0}$ has a unique solution.
- The equation $A\boldsymbol{x} = \mathbf{0}$ has infinitely many solutions.
- The equation $A\boldsymbol{x} = \boldsymbol{b}$ has a solution for all $\boldsymbol{b}$ in $\boldsymbol{R}^3$.
- There is a $\boldsymbol{b}$ in $\boldsymbol{R}^3$ such that the equation $A\boldsymbol{x} = \boldsymbol{b}$ does not have a solution.
- The columns of A span $\boldsymbol{R}^3$.
- The columns of A are linearly independent.
- The rows of A span $\boldsymbol{R}^5$.
- The rows of A are linearly independent.

11. Let U be an $n \times n$ orthogonal matrix. Which of the following statements must be true? Mark all that apply.

- [ ] U has orthonormal rows.
- [ ] The columns of U are a basis for $\boldsymbol{R}^n$.
- [ ] U is orthogonally diagonalizable.
- [ ] $UU^T = I_n$, where I_n is the $n \times n$ identity matrix.
- [ ] $nullity(U) = n$
- [ ] If λ is an eigenvalue of U, then $|\lambda| = 1$.
- [ ] $\boldsymbol{x} \cdot \boldsymbol{y} = (U\boldsymbol{x}) \cdot (U\boldsymbol{y})$ for all $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\boldsymbol{R}^n$.
- [ ] $(row(U))^{\perp} = \boldsymbol{R}^n$

12. Let S be a subspace of $\boldsymbol{R}^3$ and let $\boldsymbol{y}$ be a vector given by

$$S = span\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\} \quad \text{and} \quad \boldsymbol{y} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}.$$

Find the orthogonal projection $proj_S\, \boldsymbol{y}$ of the vector $\boldsymbol{y}$ onto S.

- ( ) $proj_S\, \boldsymbol{y} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$
- ( ) $proj_S\, \boldsymbol{y} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$
- ( ) $proj_S\, \boldsymbol{y} = \begin{bmatrix} 5/2 \\ 7/2 \\ 0 \end{bmatrix}$
- ( ) $proj_S\, \boldsymbol{y} = \begin{bmatrix} 3/2 \\ 5/2 \\ -2 \end{bmatrix}$
- ( ) $proj_S\, \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$

3. Question Details Problems 13-20 [4646148]

Problems 13-20

13. The set of all vectors lying on the line $y=-2x$ is a subspace of $\boldsymbol{R}^2$.

- True
- False

14. If A is an $n\times n$ matrix with $det(A)=0$, the columns of A are a basis for $\boldsymbol{R}^n$.

- True
- False

15. If A is similar to B, then A^T is similar to B^T.

- True
- False

16. The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is orthogonally diagonalizable.

- True
- False

17. Let $S = span\left\{\boldsymbol{w}_1, \boldsymbol{w}_2\right\}$ be a subspace of $\boldsymbol{R}^4$. If $\boldsymbol{v} \cdot \boldsymbol{w}_1 = 0$ and $\boldsymbol{v} \cdot \boldsymbol{w}_2 = 0$, then $S^\perp = span\left\{\boldsymbol{v}\right\}$.

- True
- False

18. The set $\left\{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right\}$ is an orthogonal set.

- True
- False

19. Let S be a subspace of $\boldsymbol{R}^n$, and suppose that $\boldsymbol{v}$ is in both S and $S^\perp$. Then $\boldsymbol{v} = \boldsymbol{0}$.

- True
- False

20. If S is a subspace of $\boldsymbol{R}^n$ and $\boldsymbol{y}$ is a vector in S, then $proj_S\, \boldsymbol{y} = \boldsymbol{y}$.

- True
- False

4. Question Details Problem 21 [4646156]

Problem 21

If A and B are 3×3 matrices, with $det(A)=3$ and $det(B)=6$, then $det(-2B^{-1}A^T)=$ ____.

5. Question Details Problem 22 [4646618]

Problem 22

If A is a 2×2 matrix with eigenvalue 1 and corresponding eigenvector $\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and eigenvalue -1 with eigenvector $\boldsymbol{y} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, then

$A^{101}(4\boldsymbol{x}+2\boldsymbol{y}) =$ ____ ____ .

6. Question Details Problem 23 [4646627]

Problem 23

Find the singular values of the matrix

$$\begin{bmatrix} -16 & 0 \\ 0 & 4 \end{bmatrix}.$$

$\sigma_1=$ ____

$\sigma_2=$ ____

7. Question Details Problem 24 [4646658] -

Problem 24

Let W be the subspace of $\boldsymbol{R}^4$ given by

$$W = span\left\{\begin{bmatrix}1\\2\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\0\\1\\-2\end{bmatrix}, \begin{bmatrix}0\\2\\0\\2\end{bmatrix}\right\}.$$

Find a basis for W and a basis for $W^{\perp}$. What are the values of $dim\ W$ and $dim\ W^{\perp}$? Use the green arrows to increase or decrease the number of vectors in each basis if needed.

Basis for W { }

Basis for $W^{\perp}$ { }

$dim\ W$ =

$dim\ W^{\perp}$ =

8. Question Details Problem 25 [4646667] -

Problem 25

Find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix}2 & 1 & -3\\1 & 0 & -2\\-2 & -1 & 4\end{bmatrix}.$$

A^{-1} =

9. Question Details Problem 26 [4646680] -

Problem 26

Let S be the subspace

$$S=span\left\{\begin{bmatrix}1\\0\\1\\1\end{bmatrix},\begin{bmatrix}0\\1\\-1\\1\end{bmatrix},\begin{bmatrix}1\\1\\0\\1\end{bmatrix}\right\}.$$

Use the Gram-Schmidt procedure to find an orthogonal basis for S.

When entering the vectors below, enter any fractions using "/". For example, the fraction $\frac{2}{5}$ would be entered using "2/5".

Orthogonal basis for S =

10. Question Details Problem 27 [4646683]

Problem 27

Orthogonally diagonalize the matrix

$$A=\begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}.$$

In other words, find a diagonal matrix D and an orthogonal matrix Q such that $A=QDQ^T$.

Enter the characteristic polynomial you obtain for the matrix A here. (Enter the coefficients of the polynomial in the blanks provided.)

$p(\lambda)$ = [ ] λ^2 + [ ] λ + [ ]

Enter the eigenvalues for A in **decreasing** order (from largest to smallest).

λ_1= [ ]

λ_2= [ ]

When entering the matrices below, enter any fractions using "/" and square roots using "sqrt()". For example, the fraction $\frac{2}{\sqrt{3}}$ would be entered using "2/sqrt(3)".

Enter the Q matrix here.

Q = [ ] [ ]
[ ] [ ]

Enter the D matrix here.

D = [ ] [ ]
[ ] [ ]

11. Question Details Problem 28 [4646685]

Problem 28

Find a singular value decomposition of the matrix

$$A=\begin{bmatrix} 2\sqrt{2} & 2\sqrt{2} \\ 0 & 0 \\ \sqrt{2} & -\sqrt{2} \end{bmatrix}.$$

As part of the process of finding an SVD for A you must compute an intermediate matrix B from which to compute the singular values. Enter the matrix B below. Use the green arrows to resize B if necessary.

B = [ ] [ ]
[ ] [ ]

Enter the characteristic polynomial you obtain for B here. (Enter the coefficients of the polynomial in the blanks provided.)

$p(\lambda)$ = [ ] λ^2 + [ ] λ + [ ]

Enter the singular values for A in decreasing order (from largest to smallest).

σ_1= [ ]

σ_2= [ ]

When entering the matrices below, enter any fractions using "/" and square roots using "sqrt()". For example, the fraction $\frac{2}{\sqrt{3}}$ would be entered using "2/sqrt(3)". Use the green arrows to resize the matrices below if necessary.

Enter the U matrix here.

[ ] [ ]
[ ] [ ]

Enter the Σ matrix here.

[ ] [ ]
[ ] [ ]

Enter the V matrix here (**not** V^T).

[ ] [ ]
[ ] [ ]

Assignment Details

Name (AID): **Final Exam (16537454)**
Submissions Allowed: **100**
Category: **Exam**
Code:
Locked: **Yes**
Author: **Kempton, Mark (mkempton@mathematics.byu.edu)**

Feedback Settings
Before due date
Response
Save Work
After due date
Response

Last Saved: **Apr 16, 2020 10:21 AM MDT**
Permission: **Protected**
Randomization: **Assignment**
Which graded: **Last**