0/47 Questions Answered

TIME REMAINING
3 hrs 56 mins

Midterm 2

Q1 Honor Question
2 Points

I acknowledge that I will not use notes, books, calculators, internet sources, or any assistance from other individuals as I complete this exam. I will complete the exam in one sitting. I will not discuss the exam with any other class members until after the exam period is completed.

- [x] True
- [ ] False

Q2
5 Points

Let A be a 5×8 matrix with rank 2. Which of the following statements must be true? Mark all that apply.

Q2.1
1 Point

- [x] rank$(A^T) = 2$

Q2.2
1 Point

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- [ ] The nullity of A is 5.

Q2.3
1 Point

- [ ] $\mathrm{Col}(A)$ is a subspace of $\mathbb{R}^8$.

Q2.4
1 Point

- [ ] $\dim(\mathrm{Row}(A)) = 5$

Q2.5
1 Point

- [ ] $\mathrm{Null}(A)$ is a subspace of $\mathrm{Col}(A)$.

Q3
5 Points

For each part, determine whether the given subset is a subspace of $\mathbb{R}^3$.

Q3.1
1 Point

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$$\left\{ \begin{bmatrix} t \\ 3t \\ \sqrt{t} \end{bmatrix} : t \in \mathbb{R} \right\}$$

- [ ] Yes
- [x] No

Q3.2

1 Point

$$\{\mathbf{x} \in \mathbb{R}^3 : A\mathbf{x} = \mathbf{0}\}$$

- [x] Yes
- [ ] No

Q3.3

1 Point

$$\left\{ \mathbf{x} \in \mathbb{R}^3 : A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

- [ ] Yes
- [x] No

Q3.4

1 Point

$$\text{Row}(A)$$

- [x] Yes
- [ ] No

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Q3.5
1 Point

$$\left\{ \begin{bmatrix} t \\ 3t+s \\ 5s \end{bmatrix} : s, t \in \mathbb{R} \right\}$$

- [x] Yes
- [ ] No

Q4
7 Points

Mark all that are equivalent to an $n \times n$ matrix A being an invertible matrix.

Q4.1
1 Point

- [x] $\det A \neq 0$

Q4.2
1 Point

- [x] The column space of A is $\mathbb{R}^n$.

Q4.3
1 Point

- [x] The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

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Q4.4
1 Point

- [ ] There exists a λ so $\det(A - \lambda I) \neq 0$.

Q4.5
1 Point

- [ ] $A^n = \mathbf{0}$ for some n.

Q4.6
1 Point

- [x] The rows of A are linearly independent.

Q4.7
1 Point

- [ ] The nullity of A is n.

Q5
4 Points

Given the basis $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right\}$ for $\mathbb{R}^2$ and $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find $[\mathbf{x}]_{\mathcal{B}}$ (the coordinate vector of $\mathbf{x}$ with respect to $\mathcal{B}$).

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- $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$
- $\begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$
- $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$
- $\begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$
- $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

Q6

4 Points

For each of the following, determine whether it is a linear transformation.

Q6.1

1 Point

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 1 \\ 3y \end{bmatrix}$$

- Yes
- No

Q6.2

1 Point

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - y \\ 3y \end{bmatrix}$$

Choose Files No file chosen

- [x] Yes
- [ ] No

Q6.3

1 Point

$T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix $\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 4 \end{bmatrix}$.

- [x] Yes
- [ ] No

Q6.4

1 Point

The transformation that rotates any point 27 degrees counter clockwise with the origin as the center of rotation.

- [x] Yes
- [ ] No

Q7

4 Points

Which of the following matrices is the standard matrix that corresponds to the transformation that rotates any point in $\mathbb{R}^2$ 90 degrees $\textbf{clockwise}$, then stretches horizontally by a factor of 4 and stretches vertically by a factor of 3.

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- [x] $\begin{bmatrix} 0 & 4 \\ -3 & 0 \end{bmatrix}$
- [ ] $\begin{bmatrix} -4 & 0 \\ 0 & 3 \end{bmatrix}$
- [ ] $\begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix}$
- [ ] $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$
- [ ] $\begin{bmatrix} -3 & 0 \\ 0 & 4 \end{bmatrix}$
- [ ] $\begin{bmatrix} 0 & -3 \\ -4 & 0 \end{bmatrix}$

Q8
5 Points

Assume $\mathbf{v} \neq \mathbf{0}$. Which of the following is equivalent to $\mathbf{v}$ being an eigenvector of A with eigenvalue λ? (Mark all that apply.)

Q8.1
1 Point

- [ ] $\lambda\mathbf{v}$ is in $\mathrm{Row}(A)$

Q8.2
1 Point

- [x] $A\mathbf{v} = \lambda\mathbf{v}$

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1 Point

- [x] $(A - \lambda I)\mathbf{v} = \mathbf{0}$

Q8.4
1 Point

- [ ] At least one of the entries of $\mathbf{v}$ is λ.

Q8.5
1 Point

- [x] $A(5\mathbf{v}) = \lambda(5\mathbf{v})$

Q9
4 Points

Given that $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 3,$ find
$\det \begin{bmatrix} 2d & 2e & 2f \\ 2a & 2b & 2c \\ g-5a & h-5b & i-5c \end{bmatrix}.$

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- 12
- 6
- -6
- 3
- -12
- -3
- 0

Q10
4 Points

Let A and B be 3×3 matrices with $\det(A) = 2$ and $\det(B) = -3$. What is $\det(2A^T B^{-1})$?

- $-\frac{4}{3}$
- $\frac{16}{3}$
- $\frac{4}{3}$
- $-\frac{16}{3}$
- -12
- 9
- 12
- -9

Q11
7 Points

Let A and B be $n \times n$ matrices. Mark each statement True or False.

Q11.1
1 Point

Choose Files No file chosen

$\det(AB) = \det(A)\det(B)$

- True
- False

Q11.2
1 Point

If A is invertible, $\det(A) \neq 0$.

- True
- False

Q11.3
1 Point

If B is invertible, $\det(B^{-1}) = -\det(B)$.

- True
- False

Q11.4
1 Point

If $\det(A) = 0$, the columns of A are linearly independent.

- True
- False

Q11.5
1 Point

$\det(A+B) = \det(A) + \det(B)$

Choose Files No file chosen

True

False

Q11.6
1 Point

$\det(A^T) = -\det(A)$

True

False

Q11.7
1 Point

$\det(kA) = k\det(A)$

True

False

Q12
4 Points

True or False?

Q12.1
1 Point

A matrix of size $n \times n$ cannot have more than n distinct eigenvalues.

True

False

Choose Files No file chosen

Q12.2

1 Point

A matrix that has eigenvalue 0 is not invertible.

- [x] True
- [ ] False

Q12.3

1 Point

Let A be a 2×2 matrix with eigenvalues 1 and 3. Then A^3 has eigenvalues 1 and 27.

- [ ] True
- [x] False

Q12.4

1 Point

Let A be a 4×4 matrix with characteristic polynomial $(\lambda - 1)^2(\lambda + 1)^2$. Then the eigenspace corresponding to the eigenvalue 1 is two-dimensional (i.e. its geometric multiplicity is 2).

- [ ] True
- [x] False

Q13

4 Points

Which of the following vectors is an eigenvector of the matrix

$$A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix} \quad ?$$

Choose Files No file chosen

- $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

Q14

9 Points

Let

$$A = \begin{bmatrix} 2 & -1 & -2 & 0 & 3 \\ 4 & -2 & -4 & 1 & 3 \\ 4 & -2 & -4 & -4 & 18 \\ -6 & 3 & 6 & 0 & -9 \end{bmatrix}$$

- Find the rank and nullity of A
- Find a basis of the column space of A
- Find a basis of the null space of A

(Upload a scan of your work.)

Please select file(s) Select file(s)

Q15

9 Points

If A is an $n \times n$ matrix with $A^2 = I_n$, what are the possible values of $\det(A)$? Justify your answer.

(Upload a scan of your work.)

Q 14:

$$A \xrightarrow{RREF} \begin{bmatrix} 1 & -1/2 & -1 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

nonpivot columns

$\text{rank}(A) = 2$

$\text{nullity}(A) = \#\text{ nonpivot cols.} = 3$

- A basis of $\text{col}(A)$ can be chosen by taking the 1^{st} and 4^{th} column of A:

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 2 \\ 4 \\ 4 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -4 \\ 0 \end{bmatrix} \right\}$$

- Determine a basis of $\text{null}(A)$:

Solve for $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$ from $Ax = 0$.

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \end{matrix} \\ \left[\begin{array}{ccccc|c} 1 & -1/2 & -1 & 0 & 3/2 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_2 = t, \; x_3 = s, \; x_5 = u$

$x_1 - \frac{1}{2}x_2 - x_3 + \frac{3}{2}x_5 = 0$

$\rightsquigarrow x_1 = \frac{1}{2}t + s - \frac{3}{2}u$

$x_4 - 3x_5 = 0 \rightsquigarrow x_4 = 3x_5 = 3u.$

Thus,

$$\text{null}(A) = \left\{ \begin{bmatrix} \frac{1}{2}t + s - \frac{3}{2}u \\ t \\ s \\ 3u \\ u \end{bmatrix} : t, s, u \in \mathbb{R} \right\}$$

$$= \text{span} \underbrace{\left\{ \begin{bmatrix} 1/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3/2 \\ 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}}_{\text{These three vectors form a basis of null}(A).}$$

Q15:

$$\underbrace{\det(A^2)}_{= \det(A)\det(A)} = \det(I_n) = 1.$$

Thus, $\det(A)^2 = 1$.

We get $\det(A) = \pm 1$.

An example of a matrix A such that $A^2 = I_n$ and $\det(A) = 1$

is $A = I_n$.

An example of a matrix A such that $A^2 = I_n$ and $\det(A) = -1$

is

$$A = \begin{bmatrix} 1 & & & & \\ & -1 & & 0 & \\ & 0 & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}$$

Please select file(s) Select file(s)

Q16

9 Points

Compute the determinant:

$$\begin{vmatrix} 2 & 2 & 2 & 3 \\ 2 & 2 & 4 & 4 \\ 1 & 1 & 1 & 0 \\ 3 & 4 & 5 & 6 \end{vmatrix}$$

(Upload a scan of your work.)

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Q17

9 Points

Determine all the eigenvalues of the matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. To each eigenvalue, determine the corresponding eigenspace.

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Q16

$$\begin{vmatrix} 2 & 2 & 2 & 3 \\ 2 & 2 & 4 & 4 \\ 1 & 1 & 1 & 0 \\ 3 & 4 & 5 & 6 \end{vmatrix} \xrightarrow[\substack{R_4 = R_4 - 3R_3 \\ R_3 = R_3 - \frac{1}{2}R_1}]{R_2 = R_2 - R_1} \begin{vmatrix} 2 & 2 & 2 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & -3/2 \\ 0 & 1 & 2 & 6 \end{vmatrix}$$

$$\xrightarrow[R_2 \leftrightarrow R_3]{R_3 \leftrightarrow R_4} \underbrace{\begin{vmatrix} 2 & 2 & 2 & 3 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & -3/2 \end{vmatrix}}_{\text{upper triangular matrix}} = 2(1)(2)\left(-\frac{3}{2}\right) = -6$$

Q17:

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\det(A - \lambda I_3) = \begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} -\lambda & -2 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (2-\lambda)\left(\lambda^2 - 3\lambda + 2\right)$$

$$= (2-\lambda)(\lambda-1)(\lambda-2)$$

There are two eigenvalues : $\lambda_1 = 1$ (algebraic multiplicity 1),

$\lambda_2 = 2$ (algebraic multiplicity 2).

- Find the eigenspace of λ_1:

$$[A - I_3 \mid 0] = \begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t$

$$x_1 + 2x_3 = 0 \rightsquigarrow x_1 = -2t$$

$$x_2 - x_3 = 0 \rightsquigarrow x_2 = t$$

$$E(1) = \left\{ \begin{bmatrix} -2t \\ t \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \operatorname{span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

A basis of $E(1)$ is $\mathcal{B}_1 = \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$.

- Find the eigenspace of λ_2:

$$[A - 2I_3 \mid 0] = \begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{cc} x_2 = s & x_3 = t \end{array}$$

$$x_1 + x_3 = 0 \rightsquigarrow x_1 = -t$$

$$E(2) = \left\{ \begin{bmatrix} -t \\ s \\ t \end{bmatrix} : s, t \in \mathbb{R} \right\} = \text{span}\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

A basis of $E(2)$ is

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$