Name:______________________________

Student ID:____________________________

Section:______________________________

Instructor: Webb

Math 334 (Differential Equations)
Practice Exam 1

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Calculators are not allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.
- Please do not talk about the test with other students until after the last day to take the exam.

Part I: True False Questions: *Circle either **True** or **False** for each question. (3 points each)*

1. The equation $y' = \dfrac{\sin(x^2-1)y}{y^2-1}$ is separable.

   a) True   b) False

2. The initial value problem $(4-t^2)y' + 2ty = 3t^2$, $y(1) = -3$ has a unique solution in the interval $-2 < t < 2$.

   a) True   b) False

3. Every solution of the differential equation $\dfrac{dy}{dt} = y - 5$ can be written in the form $y = 5 + ce^t$ for some constant c.

   a) True   b) False

4. Suppose an object falling through the atmosphere of a certain planet is accelerated by gravity at a rate of $10.2m/s^2$. If drag acts in the upward direction at a rate proportional to the objects velocity squared then
$$\frac{dv}{dt} = -10.2 - \gamma v^2$$
is the rate of change of the objects velocity for some $\gamma > 0$.

   a) True   b) False

5. Let $y' = -(1 - y/K)$ where $K > 0$. If $y(0) > K/2$ and $y(0) \neq K$ then $\lim_{t\to\infty} y(t) = K$.

   a) True   b) False

6. The equation $y = 5e^{3t}$ is a solution of the differential equation $4y'' - 8y' + 3y = 0$.

   a) True   b) False

Part II: *Fill in the blank with the* ***best*** *possible answer. (4 points each)*

7. A differential equation with a(n) ____________________ forms an initial value problem.

8. The differential equation $y^2 + \sin(t)y' = e^{-t}$ is called a __________,__________,__________ differential equation. (Choose from the following list : 1st order, 2nd order, 3rd order, linear, nonlinear, ordinary, and partial)

9. The equation $y(t) = 3 + (y_0 - 3)e^{6t}$ is a __________ of the initial value problem

$$y' = 6y - 18;\; y(0) = y_0.$$

10. The equilibrium solution(s) of $y' = 3y(1 - \frac{y^2}{K})$ where $K > 0$ are _________________________.

11. The difference $y_1'y_2 - y_2'y_1$ is called the _________________________ of the functions y_1 and y_2.

12. The general form of a second order linear differential equation that is homogeneous is

_________________________.

Part III: *Justify your answer and show all work for full credit.*

13. Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{y^2}{x}, \quad y(1) = 2.$$

State the solution in explicit form.

14. For the following differential equation, determine (without solving) an interval on which the solution of the equation is certain to exist and be unique. Justify your answer.

$$(\ln t)y' + y = \cot t; \;\; y(1/2) = 2.$$

15. Show that the following equation is exact, then find the solution.

$$(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)y' = 0$$

16. The following differential equation is not exact. Find an integrating factor and solve the given equation.

$$\left(\frac{y^2}{x} + 1\right) + \left(2y + \frac{1}{x}\right)y' = 0$$

17. A radioactive material disintegrates at a rate proportional to the amount of material currently present. If $Q(t)$ is the amount present at time t, then $dQ/dt = -rQ$, where $r > 0$ is the decay rate.
(a) Find an expression for the amount of this material at time t.
(b) If 100mg decays to 90mg in one week, determine r.
(c) Find the time it takes for the material to decay to 1/2 its original amount.

18. Find the general solution to the differential equation $ty' + 2y = 10e^t$.

19. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 100L of a dye solution with a concentration of 2 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water going in at a rate of 3 L/min, the well-stirred mixture going out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 5% of its original value.

20. Let $dy/dt = y^2(y/2 - 3)(1 - 2y)$. Sketch the equation's phase line and use this to determine the equations critical (equilibrium) points. Also, classify each point as either asymptotically stable, unstable, or semi-stable and sketch a few approximate solutions in the ty-plane.

END OF EXAM