Plot direction fields

You can use the command **VectorPlot** to visualize a vector field. A vector field can be thought as a map of arrows: at position (x, y) is placed a vector $(u(x, y), v(x, y))$. The syntax is:

```
VectorPLot[{u[x,y],v[x,y]},{x,a,b},{y,c,d}]
```

For example, the vector field $(u, v) = (x + y, \sin(y))$ is visualized as (Figure 1):

```
VectorPlot[{x + y, Sin[y]}, {x, -2, 2}, {y, -2, 2}]
```

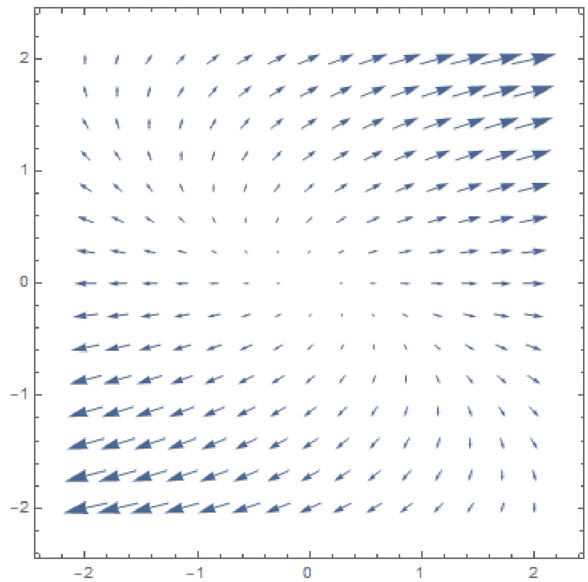

Figure 1

You can see from Figure 1 that the command VectorPlot, however, does not show exactly the vector field $(x + y, \sin(y))$. If that were the case then the vector at position $(x, y) = (2, 2)$ would be $(2 + 2, \sin 2) \approx (4, 0.91)$, which is such a long vector. The picture shows an arrow shorter than that! In fact, the command VectorPlot shows a scaled version of the vector field: Mathematica automatically scales all arrows by some appropriate factor so that they can be fitted in the box. Although this practice alters the length of arrows, it does not change their directions.

If you are using Mathematica version 11.3 or earlier, you can see that arrows in the middle of the box are much shorter than those in the top-right and bottom-left of the box, making it hard to trace curves that start from the middle of the box. A remedy is to scale each arrow by some factor to make it length-one. For example, instead of drawing vector field $(u(x, y), v(x, y))$, we draw the "normalized" vector field

$$\left(\frac{u(x,y)}{\sqrt{u(x,y)^2 + v(x,y)^2}}, \frac{v(x,y)}{\sqrt{u(x,y)^2 + v(x,y)^2}} \right)$$

For example, in the previous example:

```
u[x_, y_] := x + y
v[x_, y_] := Sin[y]
w[x_, y_] := Sqrt[u[x, y]^2 + v[x, y]^2]
VectorPlot[{u[x, y]/w[x, y], v[x, y]/w[x, y]}, {x, -2, 2}, {y, -2, 2}]
```

To draw streams directed by the arrows in a vector field, you use the command **StreamPlot**:

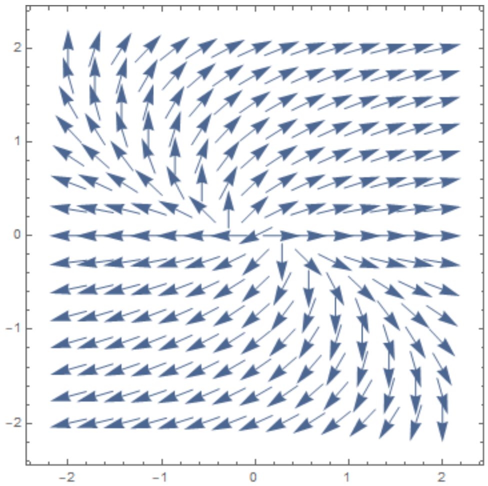

Figure 2

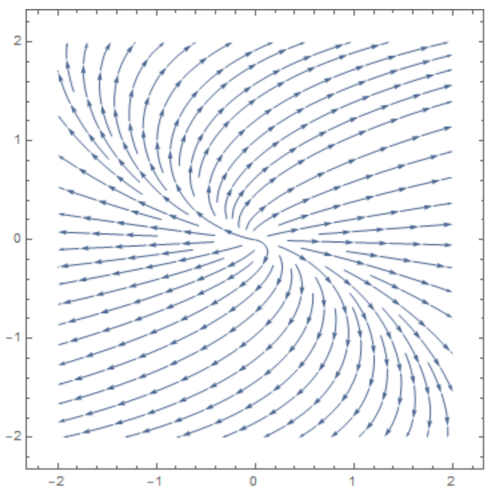

Figure 3

```
StreamPlot[{u[x, y]/w[x, y], v[x, y]/w[x, y]}, {x, -2, 2}, {y, -2, 2}]
```

Direction field of an ODE $y' = f(t, y)$ is the vector field $(1, f(t, y))$. Note that the vector $(1, f(t, y))$ has slope equal to $f(t, y)$.

Practice: *How do you draw direction field and the integral curves (i.e. the stream lines) of the ODE* $y' = -y(3 - ty)$ *in the box* $(0, 10) \times (-10, 10)$ *? You might need to normalize the direction field (as explained above) to have a better visual effect.*