Midterm II: Some problems for review

1. The rate of change of function $f(x,y) = xy + y^2$ in the direction of vector $\langle 0, 1\rangle$ at point $(2,1)$ is $\underline{\ 4\ }$. At this point, the function increases the fastest in the direction of the unit vector $\underline{\frac{1}{\sqrt{17}}\langle 1,4\rangle}$

2. A function $f(x,y)$ satisfying $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$ is said to be $\underline{\text{Continuous}}$ at (x_0,y_0).

3. Along a level set of a function, the rate of change of the function is $\underline{\ 0\ }$.

4. Let $u = \ln(1+se^t)$. Express du in terms of ds and dt. $du = u_s ds + u_t dt = \frac{e^t}{1+se^t} ds + \frac{se^t}{1+se^t} dt$

5. By Clairaut's Theorem, a smooth (i.e. infinitely differentiable) function $f(x,y)$ has at most $\underline{\ 4\ }$ different partial derivatives of third order.

6. Let $f(x,y) = ax + by$. If $\nabla f(1,1) = \langle 2, 1\rangle$ then $a = \underline{\ 2\ }$ and $b = \underline{\ 1\ }$.

7. A critical point of a function f is where $\underline{\ \nabla f\ }$ is equal to zero.

8. A function $f(x,y)$ has at most two critical points. True or false?

9. The absolute maximum over $\mathbb{R}^2$ of a function $f(x,y)$, if exists, must be attained at a critical point. True or false?

10. The absolute maximum over the square $[0,1]\times[0,1]$ of a function $f(x,y)$, if exists, must be attained at a critical point inside the square or one of the four corner points. True or false?

11. $\lim_{(x,y)\to(1,1)} \frac{2xy^2}{x^2+y^2} = \underline{\ 1\ }$ (or write DNE if the limit doesn't exist.)

12. $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2} = \underline{\ 0\ }$

13. $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \underline{\text{DNE}}$

14. Let $f(x,y) = xe^{xy}$. Find the partial derivatives of second order.

$f_{xx} = e^{xy}(2y + xy^2)$
$f_{xy} = f_{yx} = e^{xy}(2x + yx^2)$
$f_{yy} = x^3 e^{xy}$.

15. Write the equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at point $(1,-2,1)$.

16. Find the maximum and minimum values of $f(x,y) = x^2+y^2+4x-4y$ in the disc $x^2+y^2 \le 9$.

17. Find the maximum and minimum values of $f(x,y) = x^2+y^2+4x-4y$ on the circle $x^2+y^2 = 9$.

15. $8x + 4y - z + 1 = 0$

16. $\max = 9 + 12\sqrt{2}$, attained at $(x,y) = \left(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$

$\min = 9 - 12\sqrt{2}$, attained at $(x,y) = \left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$.

17. The same results as Problem 16.

18. Compute $\int\int_D \frac{y}{1+x^2} dA$ where D is the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

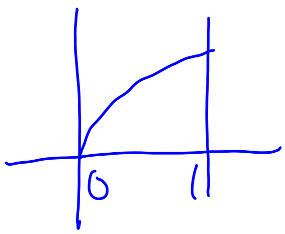
19. Compute $\int\int\int_E (x + y + z)dV$ where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

20. Write the iterated integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z)dydzdx$$

as iterated integral in the five other orders.

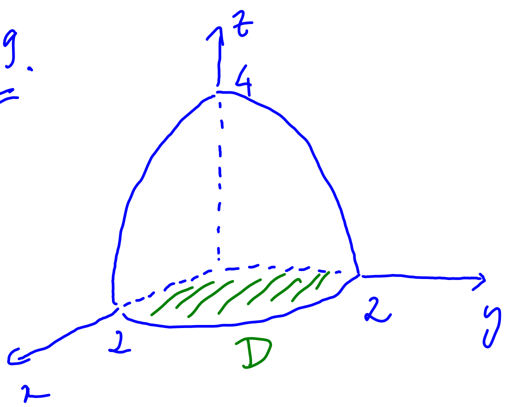
18.

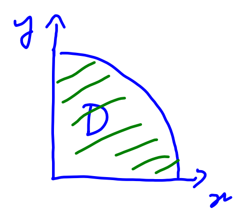


$$\int_0^1 \int_0^{\sqrt{x}} \frac{y}{1+x^2}\, dy\, dx = \int_0^1 \frac{x}{2(1+x^2)} dx$$

$$= \frac{1}{4} \ln(1+x^2)\Big|_0^1 = \frac{\ln 2}{4}$$

19.



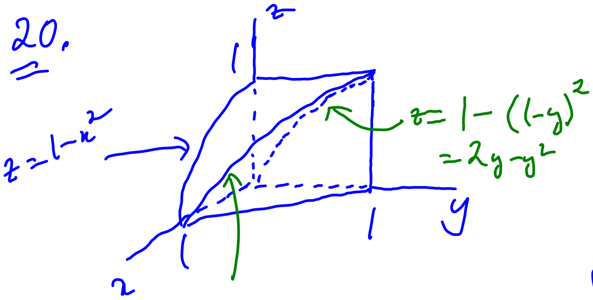


$$\iiint_E \cdots = \iint_D \int_0^{4-x^2-y^2} (x+y+z)dz\, dA = \frac{8}{15}(16+5\pi)$$

(use polar coords to integrate over D.)

You can use Mathematica to double check:
V = ImplicitRegion[x >= 0 && y >= 0 && x^2 + y^2 <= 4 && 0 <= z <= 4 - x^2 - y^2, {x, y, z}]
Integrate[x + y + z, {x, y, z} \[Element] V]
(You are not allowed to use Mathematica during the exam!)

20.



$$\begin{cases} x = x \\ y = 1 - x \\ z = 1 - x^2 \end{cases}$$

$$\int_0^1 \int_0^{1-y} \int_0^{1-x^2} \cdots \, dz \, dx \, dy \, ,$$

$$\int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} \cdots \, dx \, dy \, dz + \int_0^1 \int_{1-\sqrt{1-z}}^{1} \int_0^{1-y} \cdots \, dx \, dy \, dz$$

Here are the integrals written in two orders.

The other three are done similarly.