Midterm II: Some problems for review

1. The rate of change of function $f(x,y) = xy + y^2$ in the direction of vector $\langle 0, 1\rangle$ at point $(2,1)$ is ______. At this point, the function increases the fastest in the direction of the unit vector ______.
2. A function $f(x,y)$ satisfying $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$ is said to be __________ at (x_0,y_0).
3. Along a level set of a function, the rate of change of the function is ________.
4. Let $u = \ln(1 + se^t)$. Express du in terms of ds and dt.
5. By Clairaut's Theorem, a smooth (i.e. infinitely differentiable) function $f(x,y)$ has at most ________ different partial derivatives of third order.
6. Let $f(x,y) = ax + by$. If $\nabla f(1,1) = \langle 2, 1\rangle$ then $a =$______ and $b =$______.
7. A critical point of a function f is where _______________ is equal to zero.
8. A function $f(x,y)$ has at most two critical points. True or false?
9. The absolute maximum over $\mathbb{R}^2$ of a function $f(x,y)$, if exists, must be attained at a critical point. True or false?
10. The absolute maximum over the square $[0,1]\times[0,1]$ of a function $f(x,y)$, if exists, must be attained at a critical point inside the square or one of the four corner points. True or false?
11. $\lim_{(x,y)\to(1,1)} \frac{2xy^2}{x^2+y^2} =$ ______ (or write DNE if the limit doesn't exist.)
12. $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2} =$ ______
13. $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} =$ ______
14. Let $f(x,y) = xe^{xy}$. Find the partial derivatives of second order.
15. Write the equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at point $(1,-2,1)$.
16. Find the maximum and minimum values of $f(x,y) = x^2 + y^2 + 4x - 4y$ in the disc $x^2 + y^2 \le 9$.
17. Find the maximum and minimum values of $f(x,y) = x^2 + y^2 + 4x - 4y$ on the circle $x^2 + y^2 = 9$.

18. Compute $\int\int_D \frac{y}{1+x^2} dA$ where D is the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

19. Compute $\int\int\int_E (x + y + z)dV$ where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

20. Write the iterated integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z) dydzdx$$

as iterated integral in the five other orders.