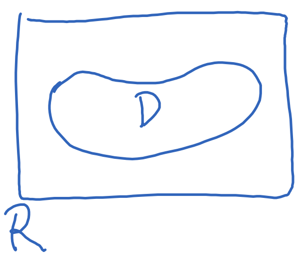
Lecture 22

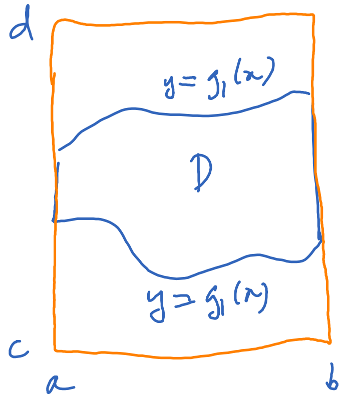
Wednesday, March 3, 2021 2:26 PM

* Prayer
* Spiritual thought
* Answering questions ....

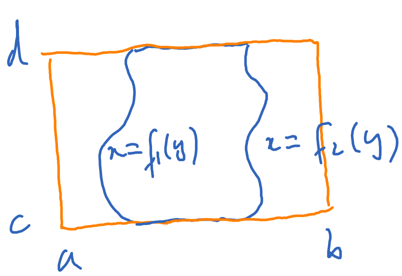
Integral over an arbitrary region :



$$\iint_D f(x,y)\,dA \overset{\text{def}}{=\!=} \iint_R \tilde{f}(x,y)\,dA$$

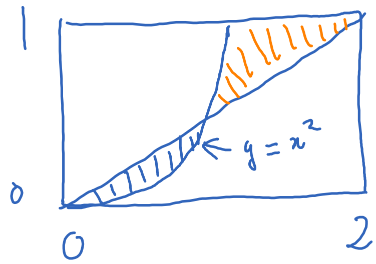


$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$



$$\iint_D f(x,y)\,dA = \int_c^d \int_{f_1(y)}^{f_2(y)} f(x,y)\,dx\,dy$$

Ex:



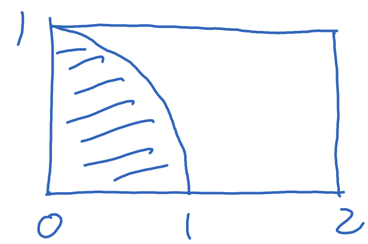
$f(x,y) = x+y \quad (g/cm^2)$

Find the mass of the shaded regions.

Double integral in polar coordinates

Depending on the region and the function...

$$\iint_D (x+y)\,dA$$



$$= \int_0^1 \int_0^{\sqrt{1-x^2}} (x+y)\,dy\,dx$$

$$= \int_0^1 \left(xy + \frac{y^2}{2} \right) \Bigg|_{y=0}^{y=\sqrt{1-x^2}} dx$$

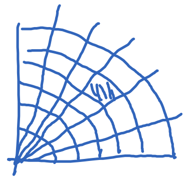
$$= \int_0^1 \left(x\sqrt{1-x^2} + \frac{1-x^2}{2} \right) dx$$

$$= \cdots\cdots$$

$2\pi \longrightarrow r^2\pi$

$\theta \longrightarrow \frac{\theta r^2 \pi}{2\pi} = \frac{\theta r^2}{2}$

D can be described as $D = \{(r,\theta) : 0 \le r \le 1,\ 0 \le \theta \le \frac{\pi}{4}\}$.

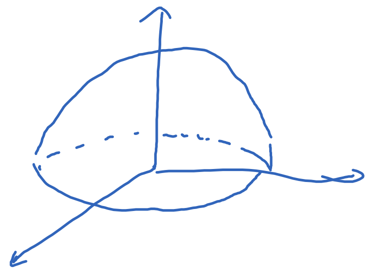


$$\iint_D f(r,\theta)\,dA = \lim \sum_{ij} f(r_i, \theta_j)\,\Delta A_{ij}$$

$$\Delta A_{ij} = \frac{(r_i + \Delta r)^2 \Delta\theta}{2} - \frac{r_i^2 \Delta\theta}{2} \approx r_i \Delta r \Delta\theta$$

$$\iint_D f(r,\theta)\,dA = \lim \sum f(r_i,\theta_j)\, r_i\, \Delta r \Delta\theta = \iint f(r,\theta)\, r\, dr\, d\theta.$$

Ex



$z = 1 - x^2 - y^2$

Find the volume of the solid.

$$\iint_D (1 - x^2 - y^2)\, dA.$$