# Lecture 28

Wednesday, March 17, 2021 2:00 PM

* Prayer
* Spiritual thought
* Answering questions....

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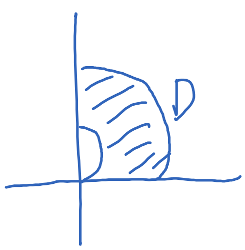
## Change of variables

$$\iint_D f(x,y)\,\underbrace{dx\,dy}_{dA} = \iint_{D'} f(x(u,v), y(u,v))\,\underbrace{\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv}_{dA}$$

Ex

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

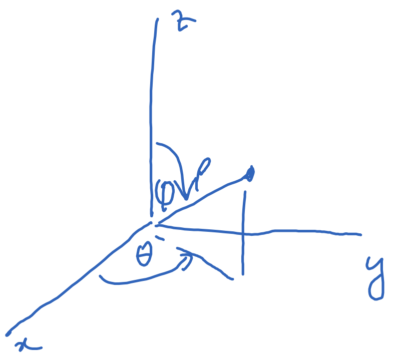
$$\frac{dx\,dy}{du\,dv} = \left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$




$$\iint_D (r+\theta)\,dA = \iint_{D'} \left(\sqrt{x^2+y^2} + \arctan\left(\frac{y}{x}\right)\right) dx\,dy$$

$$D = \{(r,\theta) : 0 \le \theta \le \frac{\pi}{2},\ \theta \le r \le \theta + 1\}$$

## Spherical coords




$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho\cos\phi \end{cases} \qquad (x, y, z) \longmapsto (\rho, \theta, \phi)$$

* Convert $(1, -\sqrt{3}, 2\sqrt{3})$ into spherical coords.

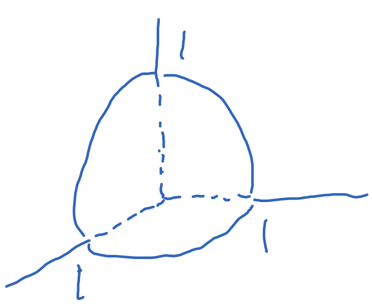
$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{16} = 4$$

$$\cos\phi = \frac{z}{\rho} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \rightsquigarrow \phi = \frac{\pi}{6}$$

$$\left.\begin{aligned} \cos\theta &= \frac{x}{\rho\sin\phi} = \frac{1}{4(\frac{1}{2})} = \frac{1}{2} \\ \sin\theta &= \frac{y}{\rho\sin\theta} < 0 \end{aligned}\right\} \theta = \frac{2\pi}{3}$$

So $(\rho, \theta, \phi) = \left(4, \frac{2\pi}{3}, \frac{\pi}{6}\right)$.

* Triple integral:




$$\iiint_E x \underbrace{dV}_{dxdydz} = \iiint_{E'} \rho \sin\phi \cos\theta \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho^3 \sin^2\phi \cos\theta \, d\rho \, d\theta \, d\phi = \cdots$$

## Vector fields

$f = x^3 + xy^2 + y^2$ $\rightsquigarrow$ potential function

$\vec{F} = \langle 3x^2 + y^2,\ 2xy + 2y \rangle$ $\rightsquigarrow$ conservative vector field.