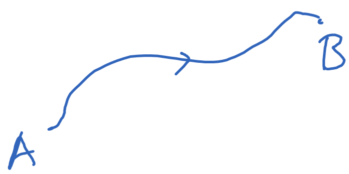
Lecture 32

Monday, March 29, 2021 2:39 PM

* Prayer
* Spiritual thought
* Answering questions ....

Fundamental theorem of Calculus for line integral:

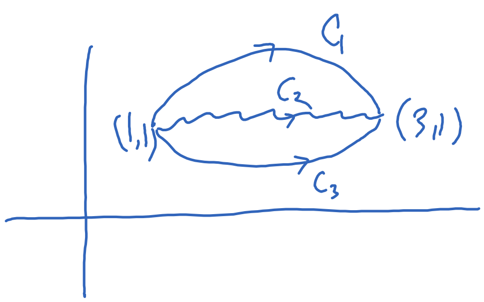
$$\int_C f_x\,dx = f(B) - f(A) \ ? \qquad (\text{No!})$$



$$\int_C \nabla f \cdot dr = f(B) - f(A)$$

Explanation: $\begin{cases} \text{chain rule} \\ df \text{ along the curve} \end{cases}$

Ex: $F(x,y) = \langle 2xy, x^2 \rangle$



$$\int_C F \cdot dr = f(3,1) - f(1,1)$$

How do we check if a vector field is conservative?

Two methods:

① $F = \langle P, Q \rangle$

$f_x = P \rightsquigarrow$ integrate over $x \rightsquigarrow f(x,y) = \dots + C(y)$

$\rightarrow f_y = \dots = Q$

② without taking antiderivatives

$F = \langle P, Q \rangle$

$$\begin{cases} P_y = Q_x \\ D \text{ is } \underbrace{\text{simply-connected}}_{\substack{\text{no holes,} \\ \text{can't have separate pieces}}} \end{cases} \Longrightarrow F \text{ is conservative}$$

Ex:

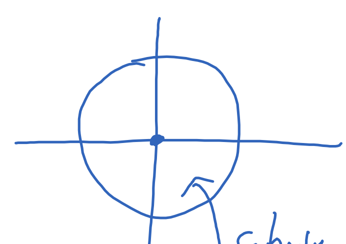
a) $\langle xy + y^2, x^2 + 2xy \rangle$

b) $\langle y^2 - 2x, 2xy \rangle$

c) $\langle y^2 e^{xy}, (1+xy)e^{xy} \rangle$

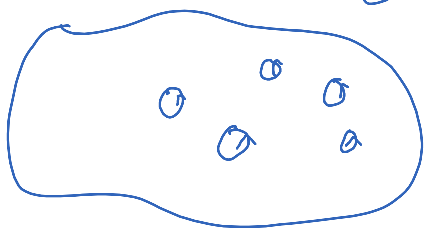
Ex: Converse is not true

$$F(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$$

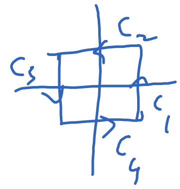


satisfies $P_y = Q_x$,
but not conservative

Green theorem



C



circulation density

$$\int_C F \cdot dr = \int_C P\,dx + Q\,dy$$

C_1: $\quad P = P(x_0, y_0 + t)$

$$Q = Q(x_0, y_0 + t)$$

$$dx = 0, \quad dy = dt$$

$$\int_{C_1} F \cdot dr = \int_{-1}^{1} Q(x_0+1, y_0 + t)\,dt$$

$$\int_{C_1} + \int_{C_3} = -\int_{-1}^{1} Q(x_0 - 1, y_0 + t)\,dt$$

$$\int_C F \cdot dr = \iint_D (Q_x - P_y)\,dA$$