Lecture 38

Monday, April 12, 2021 1:41 PM

* Prayer
* Spiritual thought
* Answering questions .....

Green's theorem:

$$\int_C F \cdot dr = \iint_D \underbrace{(Q_x - P_y)}_{\text{curl} F \cdot \vec{k}} \, dA$$

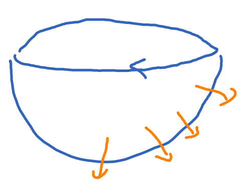
C ↑ positively oriented

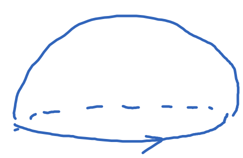
Stoke's theorem

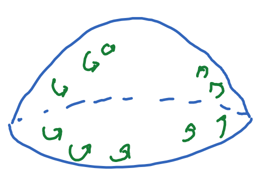
$$\int_C F \cdot dr = \iint_S \text{curl} F \cdot d\vec{S}$$

C ↑ positively oriented wrt S

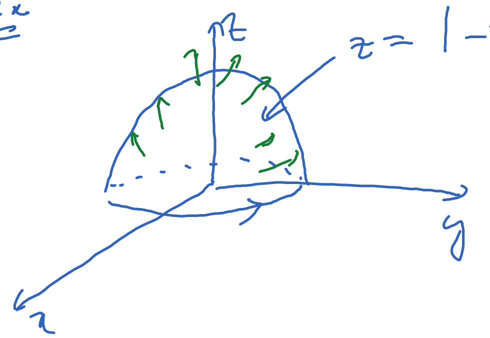
(the right hand rule)







Ex



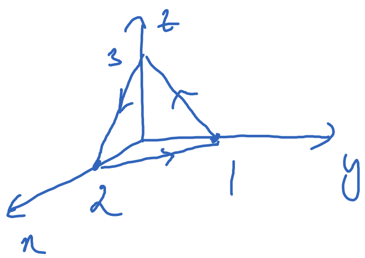
$$\iint_S \text{curl} F \cdot d\vec{S} = ?$$

where $F = \langle x, y, z \rangle$

$$\iint_S \text{curl} F \cdot d\vec{S} = \int_C F \cdot dr$$

$$\left.\begin{aligned} r(t) &= \langle \cos t, \sin t, 0 \rangle \rightsquigarrow r'(t) = \langle -\sin t, \cos t, 0 \rangle \\ F &= \langle x, y, z \rangle = \langle \cos t, \sin t, 0 \rangle \end{aligned}\right\} F \cdot r' = 0$$

Ex



$$\int_C F \cdot dr = \iint_S \text{curl}\, F \cdot d\vec{S}$$

$$S: \begin{cases} x = \\ y = \\ z = \end{cases}$$

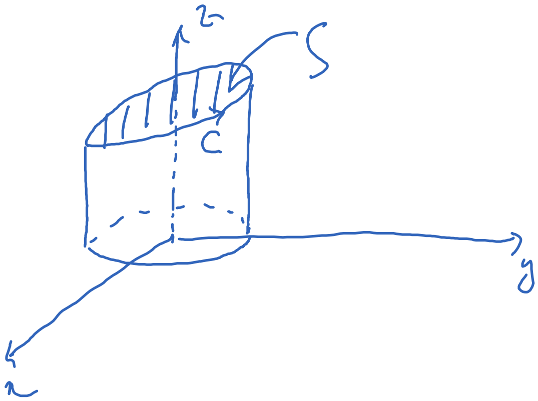
$$\left.\begin{aligned} a_1 &= \langle 2, 0, 0 \rangle - \langle 0, 0, 3 \rangle = \langle 2, 0, -3 \rangle \\ a_2 &= \langle 2, 0, 0 \rangle - \langle 0, 1, 0 \rangle = \langle 2, -1, 0 \rangle \end{aligned}\right\} \begin{aligned} n &= a_1 \times a_2 \\ &= \langle -3, -6, -2 \rangle \end{aligned}$$

normal vector: $\langle 3, 6, 2 \rangle$

$$3(x-2) + 6y + 2z = 0$$

$$\rightsquigarrow \boxed{3x + 6y + 2z = 6}$$

Ex



$$\int_C \langle 2y, xz, x+y\rangle \cdot dr$$

Ex