Midterm I: Some problems for review

The exam is 2 hours long and taken at the Testing Center between Feb 2 and Feb 4. It is a closed book exam, covering Chapter 12 and 13. No calculators are allowed. You will be provided the following formula on the exam:

$$\kappa = \frac{|r' \times r''|}{|r'|^3}, \quad \tau = \frac{(r' \times r'') \cdot r'''}{|r' \times r''|^2}, \quad N = \frac{T'}{|T'|}, \quad B = T \times N,$$

$$a_T = \frac{r' \cdot r''}{|r'|}, \quad a_N = \frac{|r' \times r''|}{|r'|}.$$

In Problems 1-10, u, v, w are vectors in 3D. Determine whether the statement is true or false. Give reason for your answers.

1) $|u + v| = |u| + |v|$

2) $|-2u| = 2|u|$

3) $|u \times v| \leq |u||v|$

4) $|u \cdot v| \leq |u||v|$

5) $|u \times v| \leq |u \cdot v|$

6) $u \cdot v = v \cdot u$

7) $u \times v = v \times u$

8) $(u \times v) \times w = u \times (v \times w)$

9) $(u \times v) \cdot u = 0$

10) The vector $\langle 3, -1, 2 \rangle$ is parallel to the plane $6x - 2y + 4z = 1$.

In Problems 11-15, $r(t)$ is a vector function of single variable. Determine whether the statement is true or false. Give reason for your answers.

11) The curve $r(t) = \langle 0, t^2, 4t \rangle$ is a parabola.

12) The curve $r(t) = \langle 2t, 3 - t, 0 \rangle$ is a curve passing through the origin.

13) $\frac{d}{dt}|r(t)| = |r'(t)|$

14) The projection of the curve $r(t) = \langle \cos 2t, t, \sin 2t \rangle$ onto the xz-plane is a circle.

15) If the curvature is equal to 0 everywhere on the curve then the curve must be a straight line.

In Problems 16-20, classify the given surfaces (cylinder/ ellipsoid/ elliptic paraboloid/ hyperbolic paraboloid/ etc).

16) In $\mathbb{R}^3$, the graph of $y = x^2$ is a/an ____________.

17) The set of points $\{(x, y, z) | x^2 + y^2 = 1\}$ is a/an ____________.

18) In $\mathbb{R}^3$, $x^2 + 4y^2 + z^2 = 1$ is the equation of a/an ____________.

19) The set of points $\{(x, y, z) | x^2 + 4y^2 - z = 0\}$ is a/an ____________.

20) The set of points $\{(x, y, z)|x^2 - 4y^2 - z = 0\}$ is a/an ______________.

Write solutions to the following problems.

21) Write the equation of the plane passing through $(2, 1, 0)$ and parallel to $x + 4y - 3z = 1$.

22) Write the equation of the plane passing through $(3, -1, 1)$, $(4, 0, 2)$, $(6, 3, 1)$.

23) Find the area of the triangle with vertices at $(3, -1, 1)$, $(4, 0, 2)$, $(6, 3, 1)$.

24) Write the equation of the plane passing through $(1, 2, -2)$ and containing the line $x = 2t, y = 3 - t, z = 1 + 3t$.

25) Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$.

26) Find the curvature of the parabola $y = x^2$ at the point $(1, 1)$.

Solution keys:

1) False
2) True
3) True
4) True
5) False
6) True
7) False
8) False
9) True
10) False
11) True
12) False
13) False
14) True
15) True
16) parabolic cylinder
17) circular cylinder
18) ellipsoid
19) elliptic paraboloid
20) hyperbolic paraboloid
21) $x + 4y - 3z = 6$
22) $-4x + 3y + z + 14 = 0$
23) $\frac{\sqrt{26}}{2}$
24) $6x + 9y - z = 26$
25) $r(t) = \langle 4\cos t, 4\sin t, 5 - 4\cos t\rangle$
26) $\frac{2}{5^{3/2}}$