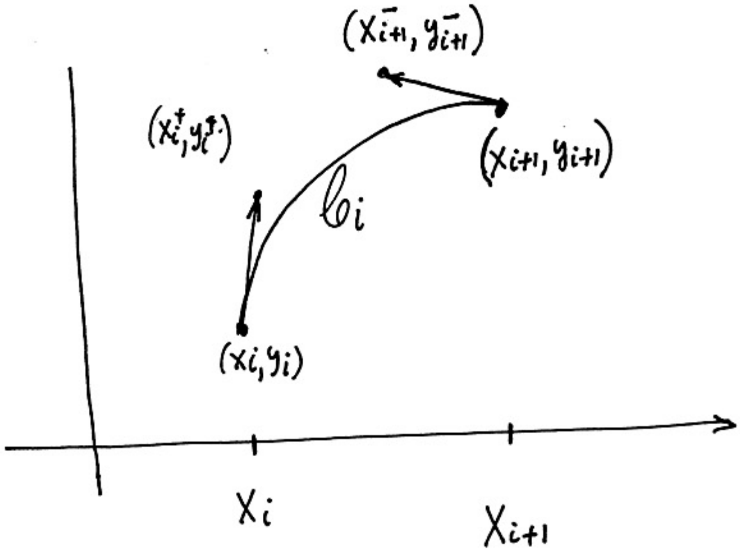
Bezier curves



Interpolation points are : (x_i, y_i), (x_{i+1}, y_{i+1})

Guidepoints are: (x_i^+, y_i^+), (x_{i+1}^-, y_{i+1}^-)

Hermite Polynomials for the parametric equations:

$$\mathcal{C}_i : \begin{cases} x_i(t) = a_0^{(i)} + a_1^{(i)} t + a_2^{(i)} t^2 + a_3^{(i)} t^3 & (1) \\ y_i(t) = b_0^{(i)} + b_1^{(i)} t + b_2^{(i)} t^2 + b_3^{(i)} t^3 & (2) \end{cases} \qquad t \in [0,1]$$

They satisfy the following BCs :

$$x_i(0) = x_i \qquad y_i(0) = y_i \qquad (3)$$

$$x_i(1) = x_{i+1} \qquad y_i(1) = y_{i+1} \qquad (4)$$

$$x_i'(0) = x_i^+ - x_i \qquad y_i'(0) = y_i^+ - y_i \qquad (5)$$

$$x_i'(1) = x_{i+1} - x_{i+1}^- \qquad y_i'(1) = y_{i+1} - y_{i+1}^- \qquad (6)$$

by multiplying by a constant all the derivatives at the ends,(*) the slopes of the tangents to the curve C_i at (x_i, y_i) and (x_{i+1}, y_{i+1}) do not change. For Bezier curves, that constant is "3". It means

$$x_i'(0) = 3\,(x_i^+ - x_i) \quad , \qquad y_i'(0) = (y_i^+ - y_i)\,3 \quad (5.2)$$

$$x_i'(1) = 3\,(x_{i+1} - x_{i+1}^-) \, , \qquad y_i'(1) = (y_{i+1} - y_{i+1}^-)\,3 \quad (6.2)$$

Using the 8 conds. the eight coefficients are determined.

$$x_i(t) = x_i + 3\,(x_i^+ - x_i)\,t + 3\,(x_i + x_{i+1}^- - 2x_i^+)\,t^2 + \left(x_{i+1} - x_i + 3\,(x_i^+ - x_{i+1}^-)\right)t^3 \quad (7)$$

$$y_i(t) = y_i + 3\,(y_i^+ - y_i)\,t + 3\,(y_i + y_{i+1}^- - 2y_i^+)\,t^2 + \left(y_{i+1} - y_i + 3\,(y_i^+ - y_{i+1}^-)\right)t^3 \quad (8)$$

(*) what it changes is the magnitude of the tangent vector to C_i at each end.