

Example 1 The Composite Simpson's rule applied to approximate

$$\int_{1.4}^{2.0} \int_{1.0}^{1.5} \ln(x + 2y) dy dx,$$

with $n = 4$ and $m = 2$ uses the step sizes $h = 0.15$ and $k = 0.25$. The region of integration R is shown in Figure 4.18 on page 230, together with the nodes (x_i, y_j) , where $i = 0, 1, 2, 3, 4$ and $j = 0, 1, 2$. It also shows the coefficients $w_{i,j}$ of $f(x_i, y_j) = \ln(x_i + 2y_j)$ in the sum that gives the Composite Simpson's rule approximation to the integral.

The approximation is

$$\begin{aligned} \int_{1.4}^{2.0} \int_{1.0}^{1.5} \ln(x + 2y) dy dx &\approx \frac{(0.15)(0.25)}{9} \sum_{i=0}^4 \sum_{j=0}^2 w_{i,j} \ln(x_i + 2y_j) \\ &= 0.4295524387. \end{aligned}$$

Figure 4.18

