Fall 2009 **Project** **Math 410**

Introduction to Numerical Methods

V. Villamizar **Due date: Tuesday November 24, 2009 (in class)**

Instructions:

i. This project consists of some applications of the methods studied in class and some programming parts. You can use any programming language of your choice (but I recommend you to use MATLAB). Students may team up with another student (Teams of two students only) and turn in only one report. **You should turn in not only the results but also your codes.**

ii. Your project will be graded carefully and should therefore, be written carefully. You will be expected to communicate your answers clearly and give logically coherent justification for them.

1. Construct a **continuously differentiable function**, $f(x)$ defined on the interval [0,10] such that

$$f(x) = \begin{cases} -x^2 + 10, & 0 \le x < 1; \\ H(x), & 1 \le x \le 4 \\ 2\,e^{x-4}, & 4 \le x \le 6 \\ S(x), & 6 \le x \le 10. \end{cases}$$

Proceed according to the following requirements and steps:

a) Construct a cubic Hermite polynomial $H(x)$ interpolating the points $(1, 9)$ and $(4, 2)$. Obtain the polynomial **explicitly**.

b) Construct a clamped cubic spline $S(x)$ interpolating function for the three points $(6, 2\,e^2)$, $(8, 1)$ and $(10, 5)$ with the additional condition $f'(10^-) = 0$. Obtain the cubic polynomials **explicitly** in each subinterval of the interval $[6, 10]$. Also, **show the tridiagonal system** employed to compute the coefficients of the cubic polynomials.

c) Evaluate your function at the points $x = 2, 7, 9$.

d) Draw the complete graph of the required function on the interval [0,10].

2. a) For the data contained in the below table, **construct and graph** the cubic Bezier polynomials. You should **provide explicit formulas** for the parametric equations $x(t)$, $y(t)$ between consecutive points.

i	x_i	y_i	x_i^+	y_i^+	x_i^-	y_i^-
0	2.8	4	3	4.5	0	0
1	3	6	3	6	3	6
2	3	1	4	1.5	3	1
3	3	3.5	2	4	2.5	3
4	4	4	4.3	3.8	5	4.5
5	4	3			4.2	2.5

Table 1:

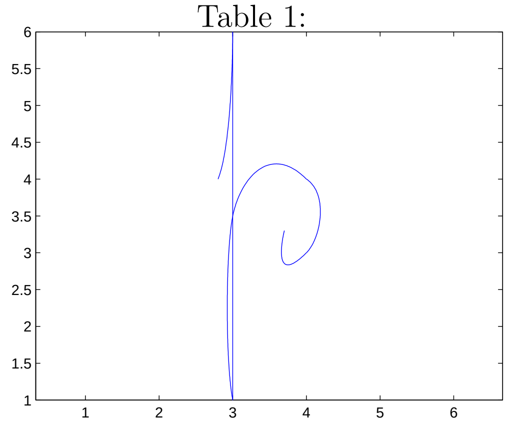

Figure 1:

b) Add an additional and final point $(3.7, 3.3)$ and define an appropriate entering guidepoint. Adjust all guidepoints to approximate the graph of the letter p in Fig. 1. **Draw an intermediate and a final graph of your approximate symbol and report all your entering and exiting guidepoints at every node.** For each approximated graph of p draw two figures: one with guide points and segments marked and another one without them.

3. a) Following Section 4.8 in your textbook (Burden-Faires) obtain a composite Trapezoidal's rule for functions $f(x, y)$ defined in two dimensional rectangular regions. Your formula should include the error term.
Hint: Try to imitate the derivation presented in pages 227-229 of your book for Simpson's rule or my class notes posted on my website.

b) Write a general program to approximate double integrals using your trapezoidal's rule of part (a).

c) Apply your program to approximate

$$\int_0^1 \int_0^1 (x^2 + y^2)\, dx\, dy$$

Indicate clearly in your program your input as well as your output parameters. Perform your computations for
i) $\Delta x = \Delta y = 0.5$, and ii) $\Delta x = \Delta y = 0.125$

d) Compare your approximate solutions with the exact solution in each case and compute the error.

e) Determine from your expression for the error, the smallest number of subintervals n (in the x-direction) and m (in the y-direction), such that $n = m$, which is necessary to approximate the integral given within 10^{-3} of its actual value.

4. a) Solve problem 24 of Section 4.3. Use $x_1 = x_0 + h$ and $x_2 = x_0 + 2h$ to obtain a_i $(i = 1, 2, 3)$. You may use MAPLE's algorithm under Linear System in my web page to understand MAPLE syntaxis for matrices and vectors. Once you obtain the weights a_i, find the factor k of the error term. Attach your MAPLE worksheet.

b) Modify Simpson algorithm (algorithm 4.1 in textbook) so that this can handle integration of a tabulated data. Write a Simpson's rule computer code that read an arbitrary data file.

c) Apply your code to the below data.

x	**y**	**x**	**y**
0	0		
0.0500	0.1571	1.0500	-0.0240
0.1000	0.3266	1.0500	-0.0240
0.1500	0.5054	1.1000	-0.4739
0.2000	0.6897	1.1500	-0.9586
0.2500	0.8752	1.2000	-1.4692
0.3000	1.0574	1.2500	-1.9949
0.3500	1.2309	1.3000	-2.5236
0.4000	1.3904	1.3500	-3.0417
0.4500	1.5302	1.4000	-3.5344
0.5000	1.6446	1.4500	-3.9862
0.5500	1.7278	1.5000	-4.3810
0.6000	1.7745	1.5500	-4.7023
0.6500	1.7795	1.6000	-4.9340
0.7000	1.7383	1.6500	-5.0607
0.7500	1.6472	1.7000	-5.0679
0.8000	1.5033	1.7500	-4.9428
0.8500	1.3048	1.8000	-4.6750
0.9000	1.0512	1.8500	-4.2562
0.9500	0.7433	1.9000	-3.6818
1.0000	0.3836	1.9500	-2.9504
		2.0000	-2.0646