# MATH 119 Chapter 11

## 0.1 Definition 1: Random Variables

Consider the following variables:

1. Distance from a bird's nest to the nearest one (meters).
2. Life time of certain machine (years).

Such quantities are called **continuous random variables**. Their values may be measured only to the nearest integer, but they actually range over an interval of real numbers.

## 0.2 Definition 2: Discrete Probability Function of a Random Variable

A function defined over a finite or an infinite discrete set of a random variable $\{x_1, x_2, \ldots x_n\}$ or $\{x_1, x_2, \ldots\}$, respectively, is a probability function if

$$\text{i)}\quad 0 \leq f(x_i) \leq 1 \qquad i = 1, 2, \ldots, \quad \text{and} \quad \text{ii)}\quad f(x_1) + f(x_2) + \ldots f(x_n) = 1$$

## 0.3 Definition 3: Probability Density Function

A function $f$ is a **probability density function** of a random variable $x$ in the interval $[a, b]$ if

$$\text{i)}\quad f(x) \geq 0 \quad \text{for all } x \text{ in} \quad [a, b] \quad \text{ii)}\quad \int_a^b f(x)\, dx = 1$$

## 0.4 Definition 4

Probability that a random variable $x$ is in an interval $[c, d]$ contained in $[a, b]$

$$P(c \leq x \leq d) = \int_c^d f(x)\, dx$$

## 0.5 Proposition 1

If the domain of the probability density $f$ is $(-\infty, \infty)$ then,

1. $P(x \leq b) = P(x < b) = \int_{-\infty}^b f(x)\, dx$
2. $P(x \geq a) = P(x > a) = \int_a^{\infty} f(x)\, dx$
3. $P(-\infty < x < \infty) = \int_{-\infty}^{\infty} f(x)\, dx$

## 0.6 Example 1

(Problem #36 in Section 11.1) Time required for a person to learn a task has a probability density function given by

$$f(x) = \frac{8}{7(x-2)^2}, \qquad x \in [3, 10]$$

Find the probabilities that a person will learn the task in
i) less than 4 minutes
ii) more than 5 minutes.

## 0.7 Definition 5: Mean or Expected Value for Random Variable

For a discrete random variable

$$\mu = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \ldots x_n p(x_n) = \sum_{i=1}^{n} x_i p(x_i).$$

For a continuous random variable

$$E(x) = \mu = \int_a^b x f(x)\, dx$$

## 0.8 Definition 6: Variance and Standard Deviation for a Random Variable

The Variance and Standard Deviation measure how spread are the values of a random variable
Discrete case:

$$Var(x) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i), \qquad \sigma = \sqrt{Var(x)}$$

Continuous case:

$$Var(x) = \int_a^b (x-\mu)^2 f(x)\, dx, \qquad \sigma = \sqrt{Var(x)}$$

## 0.9 Proposition 2: Alternative Fromula for the Variance

$$Var(x) = \int_a^b x^2 f(x)\, dx - \mu^2.$$