MATH 411 Project 1

1. The Kermack-McKendrick model for the course of an epidemic in a population is given by the system of ODEs

$$y_1' = -cy_1y_2, \tag{1}$$
$$y_2' = cy_1y_2 - dy_2, \tag{2}$$
$$y_3' = dy_2, \tag{3}$$

where y_1 represents susceptibles, y_2 represents infective in circulation, and y_3 represents infectives removed by isolation, death, or recovery and immunity. The parameters c and d represent the infection rate and removal rate, respectively. Solve this system numerically with a R-K fourth order method (develop your own code) for $c = 1$ and $d = 5$, and initial values $y_1(0) = 95$, $y_2(0) = 5$, $y_3(0) = 0$. Integrate from $t = 0$ to $t = 1$. Plot each solution component on the same graph as a function of t. As expected with an epidemic, you should see the number of infectives grow at first, then diminish to zero. Experiment with other values for the parameters and initial conditions. If possible, find values for which the epidemic does not grow, or for which the entire population is wiped out.

2. (a) Write a routine based on Newton's method to solve the system on nonlinear equations

$$(x_1 + 3)(x_2^3 - 7) + 18 = 0 \tag{4}$$
$$\sin(x_2 e^{x_1} - 1) = 0, \tag{5}$$

with starting point $\mathbf{x}_0 = [-0.5 \;\; 1.4]^T$.

(b) Write a routine based on Broyden's method to solve the same system with the same starting point

(c) Knowing that the exact solution is $\mathbf{x}^* = [0 \;\; 1]^T$. Compute the convergence rates of the two methods by computing the error in each iteration. Perform several iterations. For each method, find average values for C and p in the equation $||\mathbf{x}^k - \mathbf{x}^*||_2 = C||\mathbf{x}^{k-1} - \mathbf{x}^*||_2^p$. For this use a log-log scale and least squares approximation.

(d) How many iterations does each method require to attain full machine precision?