in regions where the exact solution is nearly linear and $u'''' \approx 0$.

On finer grids the solution looks better (see Figure 2.6(c) and (d)), and as $h \to 0$ the method does exhibit second order accurate convergence. But it is necessary to have a sufficiently fine grid before reasonable results are obtained; we need enough grid points to enable the boundary layer to be well resolved.

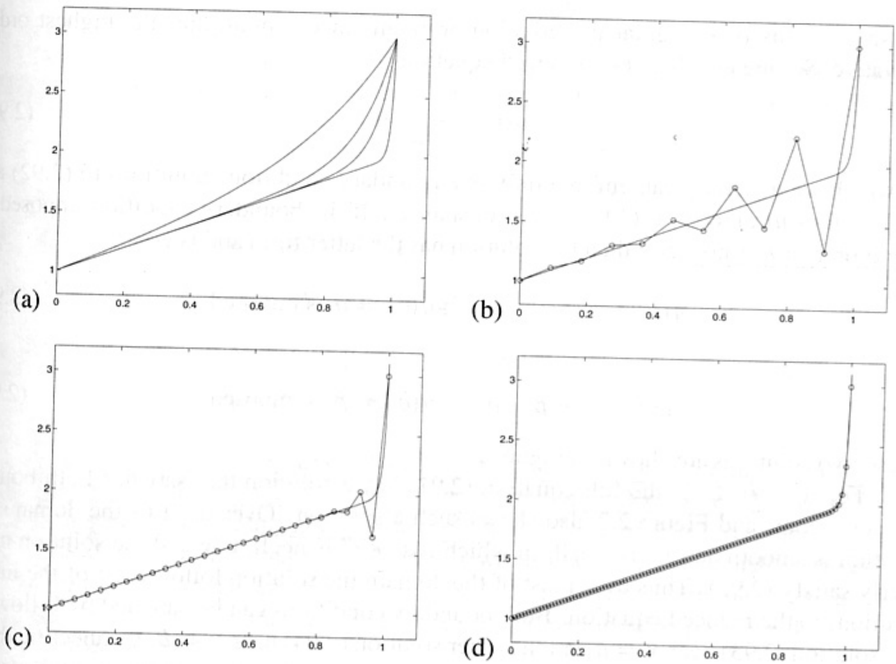

Figure 2.6. (a) *Solutions to the steady state advection-diffusion equation* (2.88) *for different values of* ϵ. *The four lines correspond to* $\epsilon = 0.3,\ 0.1,\ 0.05,$ *and* 0.01 *from top to bottom.* (b) *Numerical solution with* $\epsilon = 0.01$ *and* $h = 1/10$. (c) $h = 1/25$. (d) $h = 1/100$.

Compare with Solin for $u''(x) = 0, \quad a \to \infty$.

$u(0) = 1, \; u(1) = 3$

alue problem $u''(x) = f(x)$, as the following example illustrates.

Example 2.4. Figure 2.9 shows the error as a function of h for three methods we ave discussed on the simplest BVP of the form

$$\begin{aligned} u''(x) &= e^x \quad \text{for } 0 \le x \le 3, \\ u(0) &= -5, \quad u(3) = 3. \end{aligned} \tag{2.121}$$

he error behaves in a textbook fashion: the errors for the second order method of Secion 2.4 lie on a line with slope 2 (in this log-log plot), and those obtained with the fourth rder method of Section 2.20.1 lie on a line with slope 4. The Chebyshev pseudospecral method behaves extremely well for this problem; an error less than 10^{-6} is already

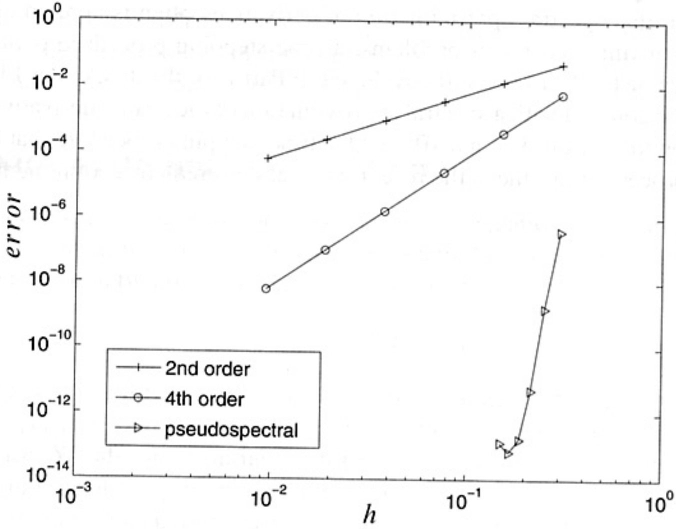

Figure 2.9. *Error as a function of h for two finite difference methods and the hebyshev pseudospectral method on* (2.121).