MATH 511

Numerical Methods for Partial Differential Equations

Winter Semester 2024

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Texts: Finite Difference Methods for Ordinary and Partial Differential Equations (PDEs) by Randall J. LeVeque, SIAM, 2007.
Professor Villamizar Own Notes in my Webpage.

OUTLINE

1. **Introduction.**

   1.1 Example of Problems Leading to Partial Differential Equations.
   1.2 Second Order Partial Differential Equations. Classification

2. **Introduction to Finite Difference Methods for Ordinary Differential Equations (ODE)**

   2.1 Derivation of Finite Difference Approximations.
   2.2 A Simple Finite Difference Method for a Linear Second Order ODE.
   2.3 Consistency, Convergence, and Stability.
   2.4 Neumann Boundary Conditions.
   2.5 Stability in the L^2-Norm.
   2.6 Nonuniform Grids and Singular Perturbation Methods.
   2.8 Spectral Methods.

3. **Introduction to Finite Element Methods for Ordinary Differential Equations**

   3.1 The Weak Form of a Boundary Value Problem (BVP).
   3.2 The Galerkin Method.
   3.3 Basis Functions, Stiffness Matrix and Load Vector.
   3.4 Piecewise Polynomials Basis and The Finite Element Method.
   3.5 Application to a One-Dimensional Steady State Heat Equation with Variable Conductivity.
   3.6 Nonhomogeneous Boundary Conditions
   3.7 Computer Implementation and Error Analysis.
   3.8 Project 1

4. **Finite Difference Methods for Elliptic Equations**

4.1The Laplace Equation
- 4.1.1 The 5-Point Stencil for the Laplacian.
- 4.1.2 Accuracy and Stability.
- 4.1.3 The 9-Point Laplacian.
- 4.1.4 The Deferred-Correction Method for the Laplacian.

4.2 The Helmholtz Equation.
- 4.5.1 Time-Harmonic Acoustic Scattering.
- 4.5.2 Finite Differences Numerical Schemes for Helmholtz Equation. High Order Deferred Correction Method.
- 4.5.3 Farfield Expansions Absorbing Boundary Conditions

4.6 Project 2

5. **Finite Difference Methods for Parabolic Partial Differential Equations.**

5.1 Explicit Finite Difference Schemes: Forward in Time and Centered in Space.
5.2 Local Truncation Error and Consistency. Convergence
5.3 Implicit Finite Difference Schemes: Backward Time – Centered in Space, Crank-Nicholson.
5.4 Stability. Matrix Stability Analysis.
5.5 The Lax Equivalence Theorem.
5.6 Neumann Boundary Conditions.
5.7 Multidimensional Parabolic Problems. ADI Methods.
5.8 Project 3

6. **Elliptic Grid Generation.**

6.1 Generalized Curvilinear Coordinates.
6.2 Amsden-Hirt Method.
6.3 Basic Iterative Methods for Linear Systems: Jacobi, Gauss-Siedel and SOR.
6.4 Winslow Grid Generation Method
6.5 Time-Harmonic Acoustic Scattering from Complexly Shaped Obstacles.
6.6 Final Project

(7. **Finite Difference Methods for Hyperbolic Partial Differential Equations.)**
(If time permits)

7.1 Advection Equation.
7.2 The Courant, Friedrichs, Lewy theorem.
7.3 Lax-Friedrichs and Lax-Wendroff Methods.
7.4 Multidimensional Hyperbolic Problems.

Course Objectives: This course is designed to prepare students to solve mathematical problems modeled by partial differential equations that cannot be solved directly using standard mathematical techniques, but which are amenable to a computational approach. Students are introduced to the discretization methodologies, with particular emphasis on the Finite Differences Method that allows the construction of accurate and stable numerical schemes. In depth discussion of theoretical aspects such as stability, analysis, and convergence will enhance the students' understanding of the numerical methods. Students will also be required to perform programming and computation to gain experience in implementing the schemes and to be able to understand the numerical performance of the various numerical methods.

I believe that my role as your instructor is to foster your learning process of mathematics. I will do my best to fulfill this role. I know that we will enjoy this class as we go along by making a consistent effort throughout the semester. **My best advice to you is found in D&C 4:2 replacing the first line by …. O ye that embark in Math 511, see that ye work with all ….**

The evaluation for this course consists of Homework, a Midterm Exam, Projects (3), and a Final Project.
Midterm Exam: Thursday March 7 -- Friday March 8 at the Testing Center.

Homework: Homework will consist of some theoretical questions and applications of the numerical methods learned in class to some initial and/or boundary value problems. They will be more like small projects. Usually, you will need to use a computer to implement the algorithms needed for the homework. Programming is an important part of this class. It will give you a wonderful opportunity to learn more and develop your programming skills. I strongly recommend that you use MATLAB as your programming language, but you can also use Python, FORTRAN, JAVA, or C++.
There are a number of MATLAB written codes in **Leveque's Homework website: http://www.amath.washington.edu/~rjl/fdmbook.**

Grading: Grades will be based on cumulative points earned as follows:
Homework 20%, Midterm Exam 15%, Projects (3) 15% each.
and Final Project 20%

Keep in mind that a good grade is the result of a good learning process. All of you can get a good grade by successfully experiencing this learning process.

Honor Code: In keeping with the principles of the BYU Honor Code, students are expected to be honest in all of their academic work. Academic honesty means, most fundamentally, that any work you present as your own must in fact be your own work and not that of another. Violations of this principle may result in a failing grade in the course and additional disciplinary action by the university. Students are also expected to adhere to the Dress and Grooming Standards. Adherence demonstrates respect for yourself and others and ensures an effective learning and working environment. It is the university's expectation, and my own expectation in class, that each student will abide by all Honor Code standards. Please call the Honor Code Office at 422-2847 if you have questions about those standards.

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