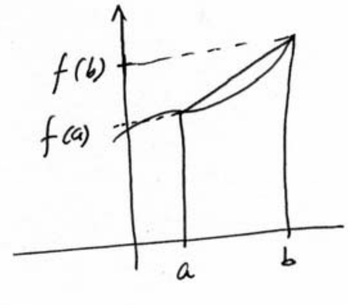
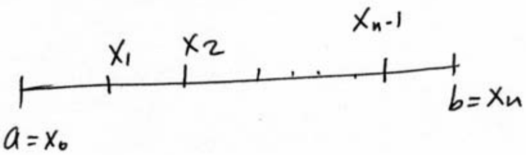
Numerical Solution of Fredholm Int. Equs.

Review Trapezoidal rule:

$$\int_a^b f(x)\,dx \simeq \frac{b-a}{2}\left[\frac{f(b)+f(a)}{2}\right]$$



If



$x_i - x_{i-1} = h$ uniform partition.

then,
$$\int_a^b dx = \int_{a=x_0}^{x_1} dx + \int_{x_1}^{x_2} dx + \cdots + \int_{x_{n-1}}^{x_n} dx \simeq$$

$$\simeq \frac{h}{2}\left[f(a) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(b)\right]$$

$$= \frac{h}{2}\left[f(a) + \sum_{j=1}^{n-1} f(x_j) + f(b)\right].$$

Now, Consider $Ku - \lambda u = f$

or
$$\boxed{\int_a^b K(x,y)\,u(y)\,dy - \lambda u(x) = f(x).} \qquad (28.1)$$

Make a uniform partition of interval $[a,b]$.

Evaluate the integral equation at these points. $x = x_i$, $i = 0, 1, 2, .., n$

$$\int_a^b K(x_i, y)\, u(y)\, dy - \lambda u(x_i) = f(x_i), \quad i = 0, 1, 2, .., n$$

Use notation
$$f(x_i) \longrightarrow f_i$$
$$u(x_i) \longrightarrow u_i$$

$$\int_a^b K(x_i, y)\, u(y)\, dy - \lambda u_i = f_i, \quad i = 0, 1, 2, .., n$$

$n+1$ equs.

Using trapezoidal for the integral $\quad h = x_i - x_{i-1}$, $\quad h = \dfrac{b-a}{n}$

$$\int_a^b K(x_i, y)\, u(y)\, dy \simeq$$

$$\simeq \frac{h}{2}\left[K(x_i, a)\, u(a) + \sum_{j=1}^{n-1} K(x_i, x_j)\, u_j + K(x_i, b)\, u(b)\right]$$

Therefore, the integral equ. (28.1) reduces to an algebraic linear system of equations: of $(n+1)$ equs. with $(n+1)$ unkws.

$$\boxed{\frac{h}{2}\left[K(x_i, a)\, u(a) + \sum_{j=1}^{n-1} K(x_i, x_j)\, u_j + K(x_i, b)\, u(b)\right] - \lambda u_i = f_i}$$

$i = 0, 1, 2, .., n.$

Example: Use the trapezoidal rule to reduce equ.

$$\int_0^1 e^{x+y}\, u(y)\, dy - \lambda u(x) = f(x)$$

to an algebraic system of (4) equs. with (4) unknws.

$0 = x_0 \qquad \frac{1}{3} = x_1 \qquad \frac{2}{3} = x_2 \qquad 1 = x_3$

$h = \frac{1}{3}$

$x_0 = 0$

$i=0$:
$$\frac{1}{6}\left[e^{0+0} u_0 + e^{0+1/3} u_1 + e^{0+2/3} u_2 + e^{0+1} u_3 \right] - \lambda u_0 = f_0$$

$i=1$:
$$\frac{1}{6}\left[e^{\frac{1}{3}+0} u_0 + e^{1/3+1/3} u_1 + e^{1/3+2/3} u_2 + e^{1/3+1} u_3 \right] - \lambda u_1 = f_1$$

$\vdots$

$i=3$:
$$\frac{1}{6}\left[e^{1+0} u_0 + e^{1+1/3} u_1 + e^{1+2/3} u_2 + e^{1+1} u_3 \right] - \lambda u_3 = f_3$$

After solving this linear system of 4 equs. 4 unknws, we will have approximations for $u(x)$ at the nodes

x_0, x_1, x_2, x_3

$$u(x_0) = u(a) \simeq u_0$$
$$u(x_1) \simeq u_1$$
$$u(x_2) \simeq u_2$$
$$u(x_3) \simeq u_3.$$