ALGEBRAIC TOPOLOGY QUALIFYING EXAM

September 22, 2001

Clearly state which theorems you are using. Don't assume so much that the proof becomes trivial. Do as many as you can.

1. Prove that CW-spaces are normal.

2. Show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$.

3. Show that there is no simplicial embedding of a non-orientable surface in S^3.

4. Prove that if $0 \to \mathcal{A} \to \mathcal{B} \to \mathcal{C} \to 0$ is an exact sequence of chain complexes, then this induces a long exact sequence in the homology groups of these complexes.

5. Prove that if $A \subseteq \mathbb{R}^n$ and A is homeomorphic to an open subset of $\mathbb{R}^n$, then A is open in $\mathbb{R}^n$.

6. S^n can be obtained from S^{n-1} by gluing 2 copies of B^n to S^{n-1} along their boundaries. Taking the nested union of S^i, $i \in \mathbb{N}$, we obtain the CW-complex S^∞ with two cells in each dimension and with the n-skeleton of S^∞ being S^n. Prove that S^∞ is contractable.

7. Prove that for CW space X, $\tilde{H}_n(SX) \cong \tilde{H}_{n-1}(X)$ where SX is the suspension of X.