Math 113H-203	Test 4	Name ______________________
12–13 April 2002	Show relevant work!	Row ______________________
D. Wright	No calculators!	5 points each

1. Evaluate the following limits. Justify each answer.

   (a) $\displaystyle\lim_{x\to 0} \frac{x}{x+\cos x}$

   (b) $\displaystyle\lim_{n\to\infty} \frac{n!}{n^n}$

   (c) $\displaystyle\lim_{n\to\infty} \frac{\ln(1-1/n)}{\sin(1/n)}$

2. Evaluate the improper integrals that converge.

   (a) $\displaystyle\int_1^\infty \frac{dx}{x^2}$

   (b) $\displaystyle\int_0^1 \frac{1}{\sqrt[3]{x}}dx$

   (c) $\displaystyle\int_0^\infty \frac{e^x}{1+e^{2x}}dx$

3. Define what it means for the series $\displaystyle\sum_{k=0}^{\infty} a_k$ to converge to L; i.e., define L in the expression $\displaystyle\sum_{k=0}^{\infty} a_k = L$.

4. Show that $\sum_{k=1}^{N} \frac{1}{k} > 1000$ if $N > e^{1000}$. Hint: Think of the Integral Test.

5. Let $\sum a_k$ be a series with positive terms such that $(a_k)^{1/k} \to r$ as $k \to \infty$. Show that $\sum a_k$ converges if $r < 1$.

6. Find the sum $1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$.

7. Show that the Ratio Test gives no information if the limit ratio is 1; i.e., give two series so that the limit ratio for each is one, but one converges and the other does not.

8. If $\sum_{k=1}^{\infty} a_k = L$, show that $\lim_{k\to\infty} a_k = 0$.

9. For each of the following series, determine if the series converges absolutely, converges conditionally, or diverges. Justify your answer.

   (a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

   (b) $\sum_{k=1}^{\infty} \frac{\sqrt{k^3}}{k^2+1}$

(c) $\sum_{k=1}^{\infty} \frac{2^k}{k!}$

(d) $\sum_{k=2}^{\infty} \frac{(-1)^k}{(\ln k)^k}$

(e) $\sum_{k=1}^{\infty} \frac{(-1)^k (k+1)}{4k+1}$

10. Find the Taylor series for the following functions about x = 0. Write out the first five nontrivial terms of each series.

(a) $\frac{x^2}{1-x}$

(b) xe^{x^2}

(c) $\tan^{-1} x$ Hint: Take the derivative.