 Assignment 10: Due Friday 21 Mar 2008

1. Solve the PDE

\[ u_t + (x + 1)u_x = e^{-t} \]

where \( u(x, 0) = 0 \) and \( u(0, t) = t \) using the Laplace transform.

2. 
(a) Prove that differentiation switches even functions to odd ones, and odd functions to even ones.
(b) Prove the same for integration provided that we ignore the constant of integration.

3. Consider the Hamiltonian

\[ H = \sqrt{p_r^2 + \frac{p_\theta^2}{r^2}} + \frac{m^2}{2} - \frac{e^2}{r}. \]

Treat the dynamical system by the Hamilton-Jacobi method, separating the Hamilton-Jacobi equations in polar coordinates. Find an expression for \( S \) (it may involve an integral).