Project 3
Due Thursday, November 30

In this project you will compare the function \( f(x) = \sin x \) to its Taylor polynomials (in \( x \)) \( P_1(x), P_3(x), P_5(x), P_7(x), \) and \( P_9(x). \)

**Exercise 1** Plot \( f(x) \) and \( P_n(x) \) \((n = 1, 3, 5, 7, 9)\) on the interval \([0, 5]\). When you look at this particular plot, what is the largest interval of the form \([0, b]\) on which the graph of \( P_n(x) \) is indistinguishable from the plot of \( f(x)\)? (Your answer should be five different intervals, one for each of these five values of \( n \).)

**Exercise 2** Use Taylor’s Theorem and the Lagrange form of the remainder to find intervals of the form \([0, b]\) (with \( b \) as large as you can) on which \(|P_n(x) - f(x)|\) is guaranteed to be less than or equal to 0.01. (Your answer should be five different intervals, one for each of the five values of \( n \).) By what factor did your application of Taylor’s Theorem in Exercise 2 underestimate the size of these intervals in each case?

**Exercise 3** Using a computer or calculator, approximate the largest interval of the form \([0, b]\) on which \(|P_n(x) - f(x)|\) actually is less than 0.01. (Your answer should be five different intervals, one for each of the five values of \( n \).) By what factor did your application of Taylor’s Theorem in Exercise 2 underestimate the size of these intervals in each case?

Higher order Taylor polynomials in \( x \) do not always give better approximations of \( f \) than lower order Taylor polynomials in \( x \) when \( x \) is far from 0.

**Exercise 4** For \( n = 1, 3, 5, \) and 7, approximate the smallest positive value of \( x \) for which \(|P_{n+2}(x) - f(x)| \geq |P_n(x) - f(x)|\).

No Maple worksheet is provided for this project. If you choose to use Maple to do this project, here are some hints you might find helpful:

- **Maple** commands end with semicolons and are executed by hitting the enter key.
- To evaluate \( \sin x \), type “\( \sin(x); \)” and hit the enter key. For example, to evaluate \( \sin(0.86) \), type “\( \sin(0.86); \)” and hit the enter key to see that the answer is .7578425629 (to 10 decimal places).
- The expression \( 3x^2 + 6x + 7 \) should be typed “3*x^2+6*x+7”.
- To define a function \( g \) having the formula \( g(x) = x^4 - 7x \), you can type
  \[
  \text{“} g := x -> x^4-7*x; \text{”}
  \]
  and then hit the enter key. You can then apply this function to just about anything. For example, typing “\( g(0.45); \)” and hitting the enter key will produce the output -3.10899375.
• “abs(x)” means $|x|$.

• To plot the function $f$ on the interval $[a, b]$, type “plot(f(x),x=a..b);” and hit the enter key. For example, typing “plot(abs(x),x=-1..2);” and hitting the enter key will plot $|x|$ for $x \in [-1, 2]$.

• You can plot several functions on the same set of axes. For example, typing “plot([abs(x),sin(x),g(x)],x=-1..3);” and hitting the enter key will plot $|x|$, $\sin x$, and $g(x)$ for $x \in [-1, 3]$.

• More information on various Maple commands can be found on the Help menu within Maple.