1. Simplify $3 \ln 6 + \ln 24 + \ln \frac{1}{4}$. This simplifies to $4 \ln 4 + 4 \ln 3$. Here is why.

$3 \ln 6 + \ln 24 + \ln \frac{1}{4} = 3 \ln (2 \times 3) + \ln (2 \times 2 \times 3) - \ln (2 \times 2) = 3 \ln 2 + \ln 3 + 3 \ln 2 + 3 \ln 2 - 2 \ln 2 = 4 \ln 2 + 4 \ln 3.$

Remember that $\ln (ab) = \ln a + \ln b$ and $\ln \left(\frac{a}{b}\right) = \ln a - \ln b$ and $\ln \left(a^{x}\right) = x \ln a$.

2. Complete the square to find the vertex of $y = 5x^2 + 12x + 7$.

Write this as $5 \left(x^2 + \frac{12}{5} x + \frac{36}{25} \right) - \frac{36}{5} + \frac{29}{5} = 5 \left(x + \frac{6}{5}\right)^2 - 5 \left(\frac{311}{5}\right)$ and so the vertex is $\left(-\frac{6}{5}, -\frac{311}{5}\right)$.

3. Solve for $x$ the inequality $5 < |3x - 12|$.

You must have $3x - 12 > 5$ or else $3x - 12 < -5$. This is because the inequality states that this quantity, $3x - 12$ is farther from 0 than 5. Therefore, either $3x > 17$ or $3x < 7$. Thus either $x > \frac{17}{3}$ or $x < \frac{7}{3}$. You could write the solution as $\left(\frac{17}{3}, \infty\right) \cup \left(-\infty, \frac{7}{3}\right)$.

4. Find the domain of $g(x) = \frac{1}{\sqrt{7-x^2}}$.

This makes sense exactly when $7 - x^2 > 0$. Thus the domain is $\left(-\sqrt{7}, \sqrt{7}\right)$.

5. $f(x) = x \cos 2x$ is an odd function. This is because $f(-x) = -x \cos (2(-x)) = -x \cos 2x = -f(x)$. Remember a function is odd if $f(-x) = -f(x)$ and a function is even if $f(-x) = f(x)$. Some functions are neither odd nor even. For example, $e^x$ is neither odd nor even. Some functions are odd such as $\sin x$, and some functions are even such as $\cos x$. Review this topic so you can handle all the cases which can occur. It is really pretty simple if you just remember the above definition.

6. Simplify $\frac{2-3i}{3-4i}$.

This is a complex number and so you need to write it as $a + ib$ to see what complex number it is. Multiply on the top and on the bottom by $1 + i$. This gives

$$\frac{(2-3i)(1+i)}{3-4i(1+i)} = \frac{5 - 1i}{3-4i} \times \frac{1+i}{1+i} = \frac{5 - 1i}{5} = \frac{1}{2} - \frac{1}{2}i.$$ Just remember that $i^2 = -1$.

7. In the picture, $a = 7, b = 6$, and $\theta = \frac{1}{4} \pi$. Find $c$.

This is an exercise in the very important law of cosines. Using this law,

$$c^2 = 7^2 + 6^2 - 2 \times 7 \times 6 \cos \theta = 49 + 36 - 84 \left(\frac{\sqrt{2}}{2}\right) = 85 - 42 \sqrt{2}.$$ And so $c = \sqrt{85 - 42 \sqrt{2}}$.

8. Find $\cot \left(\sin^{-1} \left(\frac{3}{11}\right)\right)$.

In the picture let $\theta = \sin^{-1} \left(\frac{3}{11}\right)$ so that the picture is labeled correctly. Then the line on the bottom has length $\sqrt{11^2 - 3^2} = 4\sqrt{7}$. It follows that $\cot \theta = \cot \left(\sin^{-1} \left(\frac{3}{11}\right)\right) = \frac{3}{4\sqrt{7}}$. Similarly $\tan \left(\sin^{-1} \left(\frac{3}{11}\right)\right) = \frac{3}{4\sqrt{7}}$ and $\sec \left(\sin^{-1} \left(\frac{3}{11}\right)\right) = \frac{41}{11}$, etc.

9. Write $\sin x$ in terms of exponential functions.

From the definition $\sinh x = \frac{e^x - e^{-x}}{2}$. In this regard, remember also that $\cosh x = \frac{e^x + e^{-x}}{2}$. Note that $\cosh (-x) = \cosh (x)$ so $\cosh$ is an even function and $\sinh (-x) = -\sinh (x)$ so $\sinh$ is an even function. You should also note that
\[ e^x = \cosh x + \sinh x. \] In general, if you have any function, \( f \) defined on \( \mathbb{R} \) you can define \( h(x) = \frac{f(x) + f(-x)}{2} \) and \( g(x) = \frac{f(x) - f(-x)}{2} \) and that \( h \) is an even function and \( g \) is an odd function while \( f = h + g \). The function, \( h \) is called the even part of \( f \) and the function, \( g \) is called the odd part of \( f \). Thus \( \cosh x \) is the even part while \( \sinh x \) is the odd part.

10. Find \( \lim_{x \to \infty} \frac{17x^3 - 3(\sqrt{x})^5 - 5}{5x^3 - 3} \).

You can divide the top and the bottom by \( x^3 \) to find the fraction equals \( \frac{17 - \frac{3}{5}}{\frac{5}{x^3}} \) and then as \( x \to \infty \), the top \( \to 17 \) and the bottom \( \to 5 \) so the limit is \( \frac{17}{5} \). It always works this way. When you have the quotient of two polynomials of the same degree and you take the limit as \( x \to \infty \), the result will always be the ratio of the coefficients of the highest powers of the two polynomials. Be sure you understand why.

11. Find \( \lim_{x \to -3} \frac{\tan(2x+6)}{\arcsin(7x+21)} \).

This is an exercise in L'Hopital’s rule. 
\[
\lim_{x \to -3} \frac{\tan(2x+6)}{\arcsin(7x+21)} = \lim_{x \to -3} \frac{2\sec^2(2x+6)}{\frac{7}{\sqrt{1-(7x+21)^2}}} = \frac{2}{7}. \]
Be sure you are able to use L’Hopital’s rule. Remember that before you use L’Hopital’s rule you must have an indeterminate form. Otherwise the rule might not give the right answer.

12. Find where \( f(x) = \frac{x^2 + 8}{x^2 + 5x + 4} \) is continuous.

By limit theorems, this happens when the denominator is not equal to zero, everywhere except at \( x = -1 \) and \( x = -4 \).

13. Pick the correct statement of the mean value theorem, the intermediate value theorem and the extreme value theorem. You need to be sure you know what each of these important theorems say.

14. Define \( f'(x) \) as a limit. Review this and be sure you understand why the derivative is defined this way. Be able to use the definition to compute a derivative. Here is a question you should consider. If the derivative exists, must it be continuous?

15. Find \( y' \) if \( y = \arcsin 7x + \arctan x \).
\[
\frac{dy}{dx} (\arcsin 7x + \arctan x) = \frac{7x^2 + \sqrt{1 - 49x^2}}{(1 - 49x^2)(1 + x^2)}.
\]
This is an exercise in the rules of differentiation. Be sure you know the rules of differentiation such as the product rule, the chain rule etc. You should be able to differentiate anything which can be written down in terms of elementary functions. If you can’t do this, you don’t know how to use the rules well enough.

16. Find \( f'(x) \) if \( f(x) = x^3 \cos x - x^2 \ln x \).
\[
\frac{df}{dx} (x^3 \cos x - x^2 \ln x) = 4x^3 \cos x - x^4 \sin x - 2x \ln x - x. \]
This is an exercise in the rules of differentiation. Be sure you know the rules of differentiation such as the product rule, the chain rule etc. You should be able to differentiate anything which can be written down in terms of elementary functions. If you can’t do this, you don’t know how to use the rules well enough.

17. Find the minimum value of the function \( f(x) = 5 + 9x^2 + \frac{2}{x} \) for \( x > 0 \). You take the derivative and set it equal to zero.
\[
\frac{df}{dx} (5 + 9x^2 + \frac{2}{x}) = 2(9x^3 \frac{1-x^{-2}}{x^2}) = 0, \]
Solution is: \( \{ x = \frac{1}{\sqrt{3}} \} \). Therefore, if there is a minimum value, it occurs when \( x = \frac{1}{\sqrt{3}} \). Plugging this in to the function gives \( 5 + 9 \left( \frac{1}{\sqrt{3}} \right)^2 + \frac{2}{\left( \frac{1}{\sqrt{3}} \right)} = 5 + 3 \left( \frac{\sqrt{3}}{3} \right)^2 \).

18. Find the local minimum values of the function, \( f(x) = x^4 - 7x^2 + 7 \). You do this the same way. The extrema: \( \{ -7, \frac{-7}{4} \} \), at \( \{ x = 0 \}, \{ x = \frac{1}{\sqrt{14}} \}, \{ x = -\frac{1}{\sqrt{14}} \} \).

19. A rectangular playground is to be enclosed by a fence and divided in 4 pieces by 3 fences parallel to one side of the playground. 1900 feet of fencing is used. Find dimensions of the playground which will have the largest total area.

You need to set this up. Draw a picture. \( 5x + 2y = 1900 \). Now you want to maximize \( xy \). Therefore, you want to maximize \( f(x) = x \left( \frac{1900-2x}{5} \right) \). Do so. Be sure you can set this up and work it.

20. Find \( \int \left( 2e^x + \frac{1}{x^2} + 7 \tan x \right) dx = 2e^x + \frac{1}{2} \ln x - 7 \ln (\cos x) + C \)

21. Find \( \int \frac{x}{2x^2} dx = -\frac{1}{2} \frac{1}{x^2} + C \)
22. Let \( f(x) = \int_{-4}^{x^2+1} \sqrt{1+t^4} \, dt \). Find \( f'(x) \). This is an easy exercise in the fundamental theorem of calculus and the chain rule. Yes, you should be able to use both theorems in a single problem. 

\[
\frac{d}{dx} \left( \int_{-4}^{x^2+1} \sqrt{1+t^4} \, dt \right) = \sqrt{1+(x^2+1)^2} \cdot 2x.
\]

Here is a similar problem which you should also be able to do. Find 

\[
\frac{d}{dx} \left( \int_{\sin x}^{\cos(x^2)} \sqrt{2+t^4} \, dt \right) .
\]

If you don’t know how to do these problems, review and learn how before you take the pretest.

23. The velocity of a car moving on the \( x \) axis is \( v(t) = t^2 - 4t - 2 \) where \( t \) is time. Find its position as a function of \( t \) given its initial position is at 4.

Let \( x \) be the position. Then \( v = \frac{dx}{dt} = t^2 - 4t - 2 \).

Hence \( \frac{t^3}{3} - 2t^2 - 2t + C \) and \( C = 4 \) because at \( t = 0 \), it is given to be at 4. Hence the position is \( \frac{t^3}{3} - 2t^2 - 2t + 4 \).

24. Find \( \lim_{x \to 0} \tan \frac{3x}{5x^3} = \frac{3}{5} \). This is another L’Hopital’s rule problem.

25. Find \( \lim_{x \to 0} (1 + 2x)^{1/x} = e^2 \). This is another L’Hopital’s rule problem.

26. Let \( f(x) = 3^{\tan x} \). Find \( f'(x) \). You need to know what \( 3^{\tan x} \) is. 

\[
\frac{d}{dx} (3^{\tan x}) = 3^{\tan x} (1 + \tan^2 x) \ln 3.
\]

Be sure you can do this sort of thing before taking the pretest.

27. What is the correct definition of continuity of a function, \( f \) at a point, \( x \)? Be sure you know what it means for a function to be continuous. By this I mean the correct definition, not worthless nonsense about not taking the pencil off the paper when you draw the graph. That is not correct! It is a stupid and misleading lie and will receive no credit.

28. Find \( \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n} \sin \left( \frac{k}{n} \right) \). The sum is a Riemann sum for the integral \( \int_{0}^{1} \sin x \, dx \). Therefore, the answer is \( -\cos (1) + 1 \). Be sure you understand Riemann sums and how they approximate an integral. This is very important. Be able to recognize a Riemann sum when you see one.